Effects of Polarisation Fading upon the Performance of Skywave Radars.

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Peter Dreisiger
Supervisor: Dr. G.F. Earl

Ionospheric Effects Group
High Frequency Radar Division
Defence Science and Technology Organisation
Salisbury, South Australia

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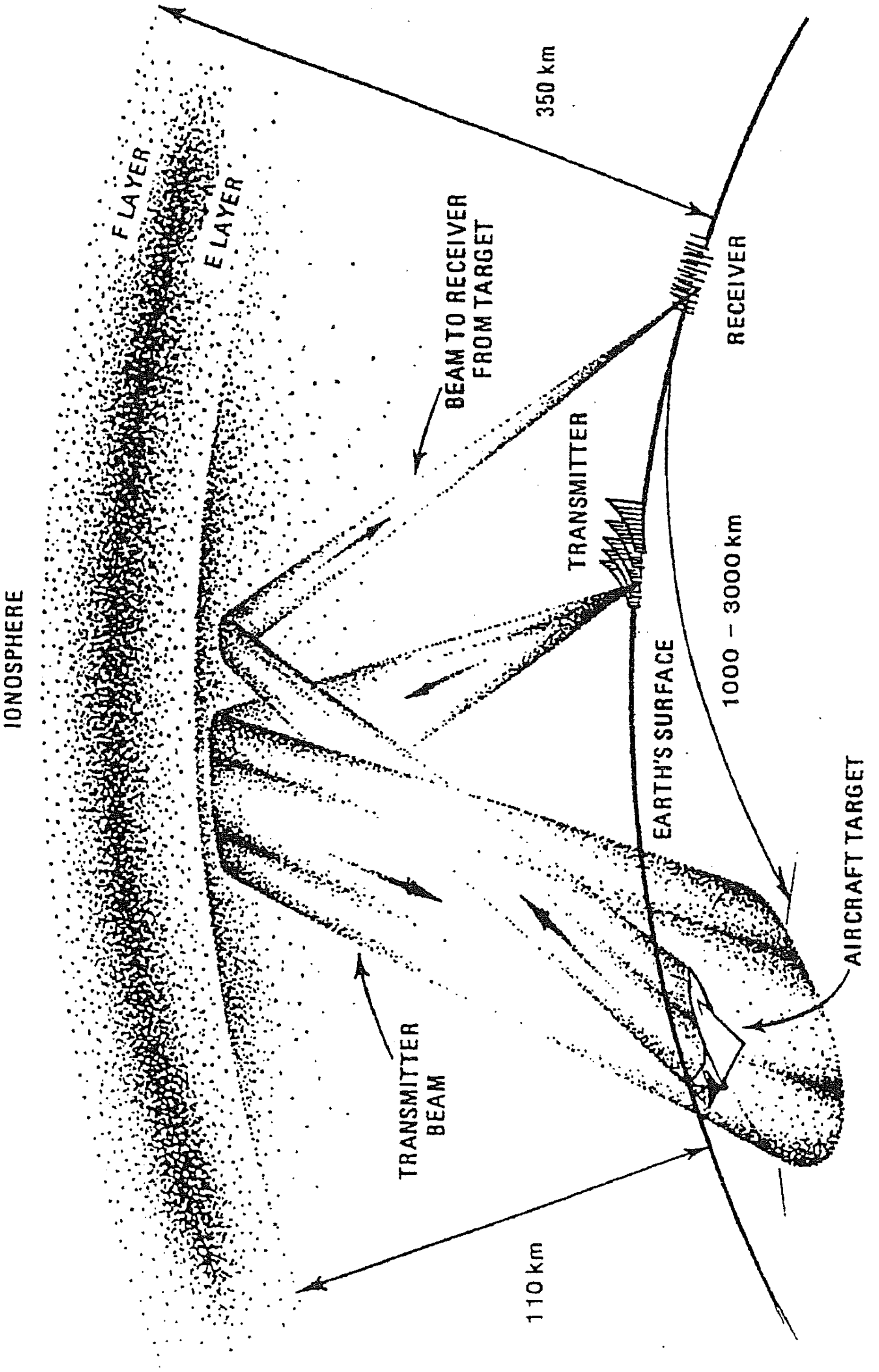


Figure 1.1 **Overview of radar geometry.**

1. INTRODUCTION

During my vacation employment at DSTO Salisbury, I was assigned to examine how Polarisation (Faraday) rotation affects Over the Horizon Radars (OTHRs) and what techniques can be used to reduce them. Like any system which relies upon HF radio waves that travel through the ionosphere, OTHR systems like Australia's Jindalee are susceptible to ionospheric fading, and thus a loss in sample resolution. While the effects of polarisation rotation are examined closely in this paper, other cause of signal fading are outlined with respect to their effects upon the operation of OTHRs.

As a target, or in this case a beacon is required before any backscatter fading can be measured, I initially investigated how critical the beacon's design parameters had to be in order to achieve the polarisation invarience required to study polarisation fading. Different designs and signal modulations were also examined to the determine what configuration would provide the greatest detectable return power.

Static models were then used to analyse how polarisation fading behaved as the operational frequencies and target ground ranges were varied and the accuracy of certain approximations were investigated. These results were then compared with ray traced values obtained after travelling ionospheric disturbances were introduced. Geographically, it is assumed in all calculations that the transmit and receive sites are located at Harts range and Mt. Everard respectively, and that the beacon is located 2000 kilometres due north unless otherwise stated.

Many thanks must go to my supervisor Dr Fred Earl for his guiding wisdom, as well as Mike Whitington for his unrivalled expertise in anything computer related, Dr Chris Coleman and everyone in IE Group for what was both an enjoyable and challenging twelve weeks.

2. PASSIVE BEACON

In order to determine the extent to which polarisation fading affects radar performance, and in particular target backscatter across all operational ground ranges, it is proposed to set up a stationary passive beacon which will act as a reflector by reradiating a percentage of the incident radar signal. The two main considerations in the design of the beacon are to maximise retransmission power and achieve polarisation invarience. Note that in this section, all propagation losses are ignored and only the effect of polarisation fading is considered.

2.1 SIGNAL MODULATION

Since the beacon is intended for stationary deployment, it lacks the Doppler shift required to uniquely identify its backscatter from the clutter received by the radar. It is thus necessary to modulate the reradiated power in such a way as to create a pseudo-Doppler shift. This can be accomplished by modulating the beacon's antenna loads so that the reradiated signal power varies in time. It has been found by Earl [1] that a square wave modulation allows optimal beacon identification, and it can be shown that a unity mark:space ratio maximises the detectable return power. Given the Fourier series transform of a square wave (equation 2.1) with duty cycle α and peak amplitude V, we have:

$$v(\theta) = V\left[\alpha + \sum_{n=1}^{\infty} \frac{2\sin n\alpha\pi}{n\pi}\cos n\theta\right] \tag{2.1}$$

Integrating over the square wave's period to get the average AC voltage, we have:

$$\begin{aligned}
V_{avg} &= \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\| v(\theta) \right\| d\theta \\
&= \frac{V}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\sum_{n=1}^{\infty} \left\| \frac{2\sin n\alpha\pi}{n\pi} \right\| \cos n\theta \right) d\theta \\
&= \frac{V}{\pi}\left[\sum_{n=1}^{\infty} \left\| \frac{2\sin n\alpha\pi}{n^2\pi} \right\| \sin n\theta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \frac{4V}{\pi^2}\sum_{n=1}^{\infty} \left\| \frac{4\sin(4n-3)\alpha\pi}{(4n-3)^2\pi^2} \right\| - \left\| \frac{4\sin(4n-1)\alpha\pi}{(4n-1)^2\pi^2} \right\| \\
\frac{\partial V_{avg}}{\partial \alpha} &= \frac{4V}{\pi}\sum_{n=1}^{\infty} \left\| \frac{4\cos(4n-3)\alpha\pi}{4n-3} \right\| - \left\| \frac{4\cos(4n-1)\alpha\pi}{4n-1} \right\| \\
&\equiv 0 \;\Leftrightarrow\; \alpha = \frac{1}{2}
\end{aligned} \tag{2.2}$$

One consequence of modulating the retransmitted signal from a stationary beacon is the decrease in average power with respect to a moving target. Given the optimum 50% duty cycle (α=0.5) square wave modulation and a voltage separation R as shown below:

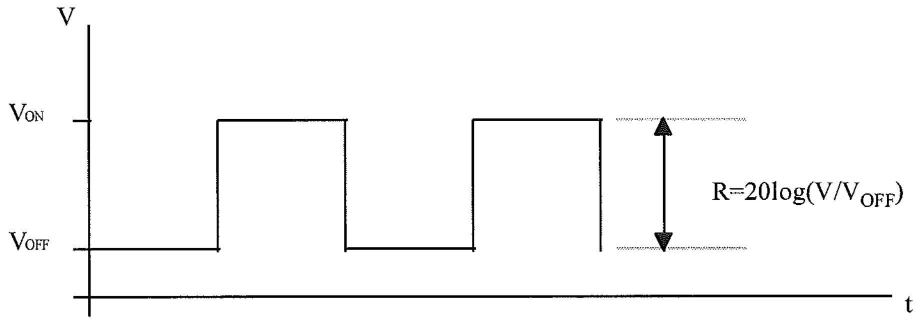


Figure 2.1. Beacon's retransmitted EMF after modulation.

we can see that the amplitude of the first harmonic (n=1 in the Fourier series expansion) is given by:

$$V_{ON} = \frac{V_{\max}}{\pi} \qquad\qquad V_{OFF} = V_{ON} \times 10^{\frac{-R}{20}}$$

If we define the modulation loss to be the signal loss of the stationary beacon with respect to a moving target, whose signal strength is V_{max}, we obtain:

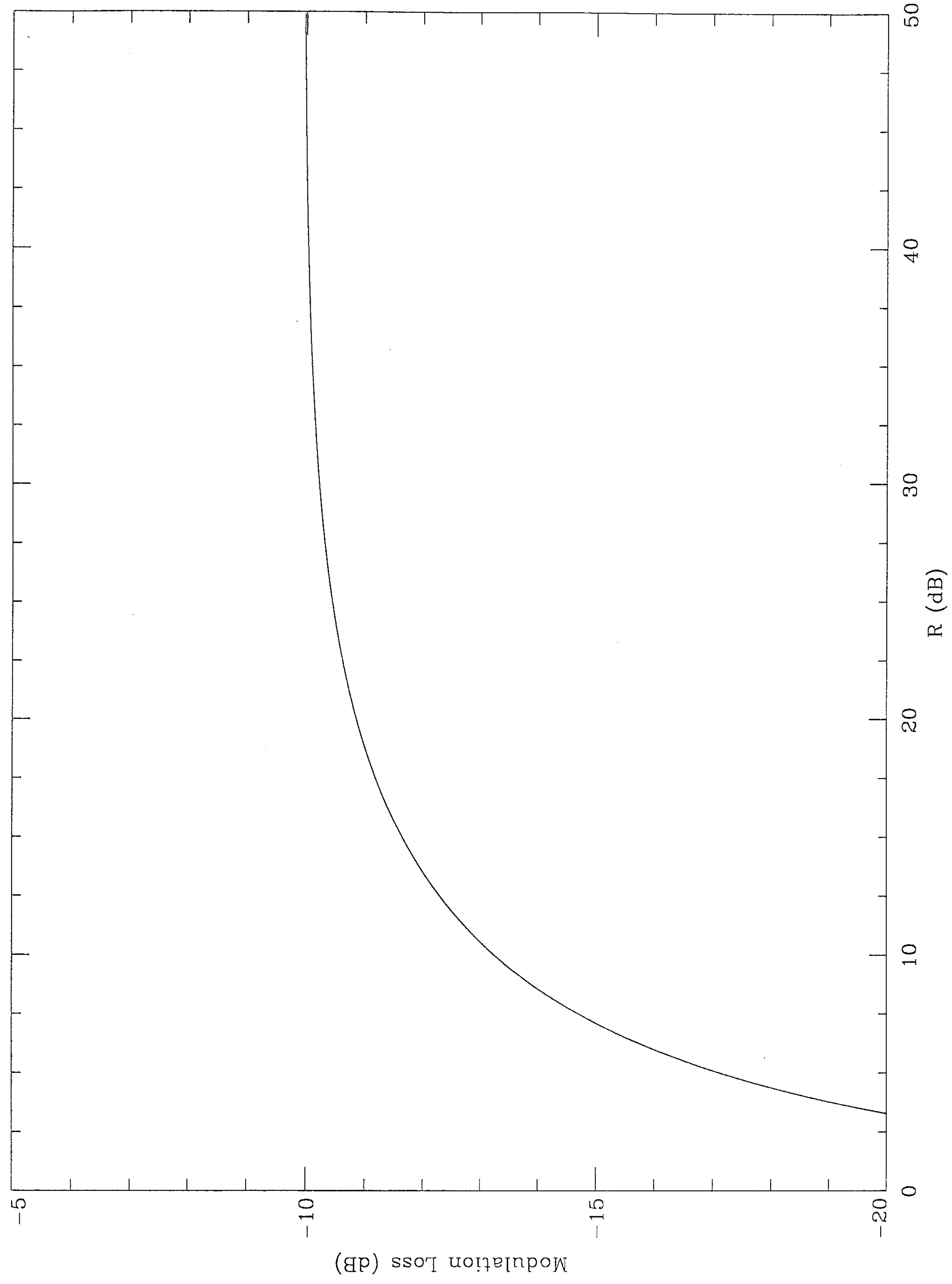
$$Modulation\ Loss = 20\log\left(\frac{1-10^{\frac{-R}{20}}}{\pi}\right) \qquad (2.3)$$

The modulation loss as shown in Graph 2.1 is plotted as R goes from 0 to 40dB. While the modulation loss tends to 9.91dB as the separation R approaches infinity, so long as the separation can be maintained at above 20dB, the additional loss will be less than 1dB.

2.2 BEACON CONFIGURATION

As we are intending to study polarisation, we require the ideal beacon's response to be independent of the angle of the incident ray's polarisation, while maximising the backscatter's signal strength. The three beacon configurations originally considered consisted of:

1. a single vertically polarised antenna
2. a horizontally and a vertically polarised antenna, electrically uncoupled, with a voltage gain ratio $\beta_B = \frac{G_H}{G_V}$
3. a horizontally and a vertically polarised antenna, coupled by a transmission line with phase delay α and a voltage gain ratio $\beta_B = \frac{G_H}{G_V}$



Graph 2.1 **Modulation losses as a function of voltage separation.**

It can be shown that for the above cases, the received EMF as a function of the transmitted EMF E, polarisation rotation on the outbound leg θ and rotation on the inbound leg ϕ is given by:

$$E_R = E\cos\theta\cos\phi \tag{2.4a}$$
$$E_R = E(\cos\theta\cos\phi + \beta_B{}^2 \sin\theta\sin\phi) \tag{2.4b}$$
$$E_R = \frac{E}{2}(\cos\theta\cos\phi - \beta_B\cos\theta\sin\phi - \beta_B\sin\theta\cos\phi + \beta_B{}^2\sin\theta\sin\phi) \tag{2.4c}$$

(see Appendix A for a complete derivation). For the passive beacon to be considered polarisation invariant under planar propagation, at least two orthogonal antennas are required, thereby eliminating case 1 whose retransmitted power would drop to zero as the incident rays approach horizontal polarisation. While both cases 2 and 3 use horizontally and vertically polarised antennas, in case 3 where the two antennas are coupled, we have a situation approximating maximum power transfer as the impedances of the two antennas are assumed to be similar. Thus under ideal circumstances, each antenna is driving a matched load and thus loses half its available voltage (or 75% of its power). As can be seen in Graph 2.4, the coupling of the horizontal and vertical antennas thus greatly affects the reradiated power as a function of polarisation rotation.

To give a qualitative idea of how each beacon configuration performs over all possible polarisation angles, plots of the total received EMF assuming no propagation losses and perfectly matched antennas (β_B=1) are shown for ϕ, $\theta\subseteq[0, 2\pi]$. As can be seen from Graphs 2.2 to 2.4, case 2 gives the maximum average reradiated power over all polarisation angles, and it is this model that is used hereafter.

For the case with the two uncoupled antennas, the sensitivity of the retransmitted power to variations in the voltage gain ratio β_B was analysed. Normalising the retransmitted power, we find that:

$$P_{Retransmitted} = E^2\left[\cos^2\theta + \beta^4\sin^2\theta\right] \tag{2.5}$$

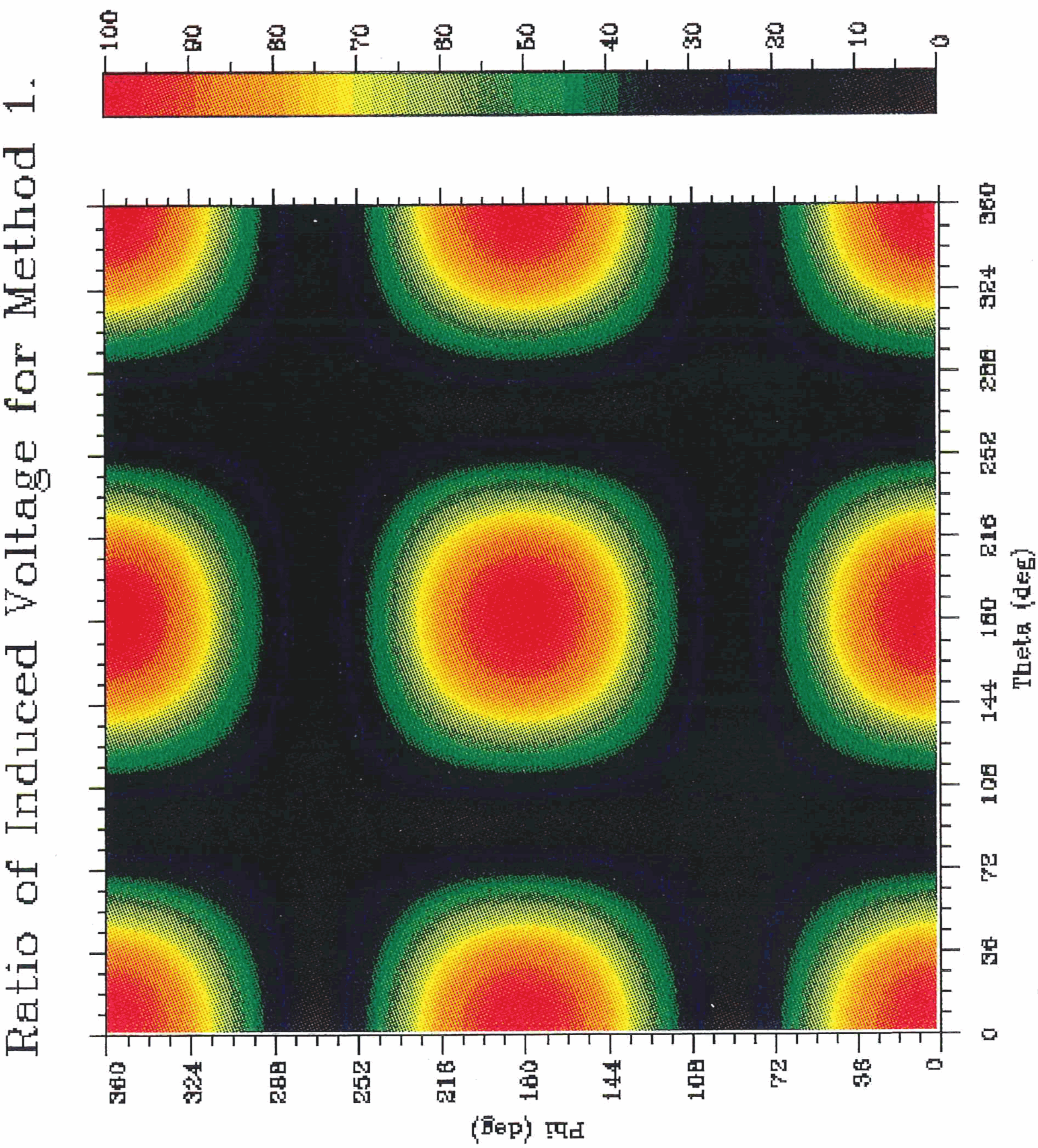
and to maximise the retransmitted power, we want β_R to approach unity (see Appendix A).

The reradiated power as β_B is varied from 0 to 3dB is shown in Graphs 2.5 through 2.7, and it is clear that by keeping the voltage gains within 1dB of each other, the power loss can be restricted to less than 1.5dB. While it is possible to match two antennas with this degree of accuracy, special care would need to be taken to ensure that the beacon's characteristics remain comparable over the operational frequency range of the radar.

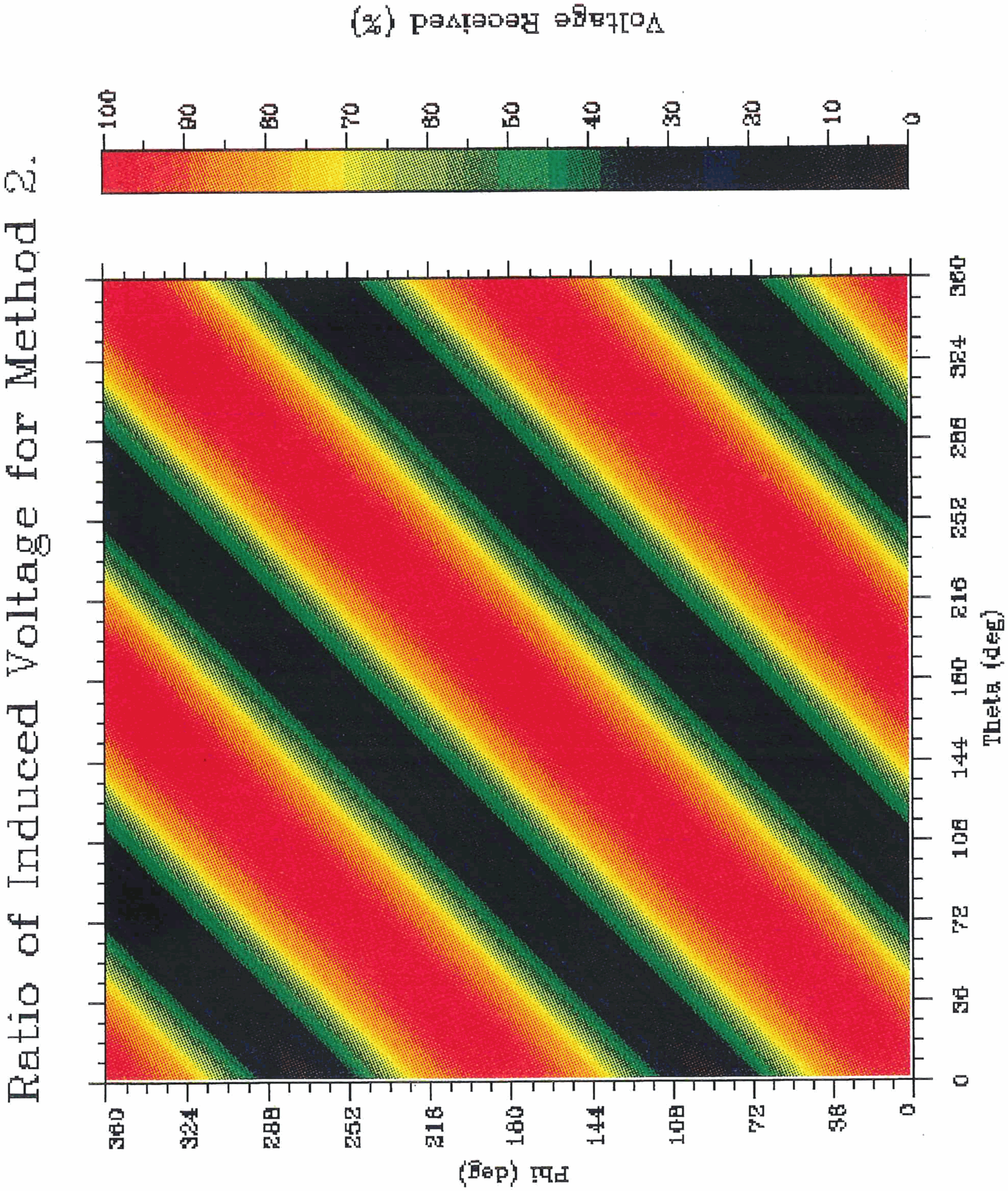
2.3 IDENTIFICATION OF BEACON BACKSCATTER

Currently, the receive array at Mt. Everard is only capable of detecting vertically polarised backscatter, thus negating most of the benefits gained from having a polarisation invariant beacon. To allow the polarisation rotation to be studied, it is necessary to measure the horizontal component of any polarised ray.

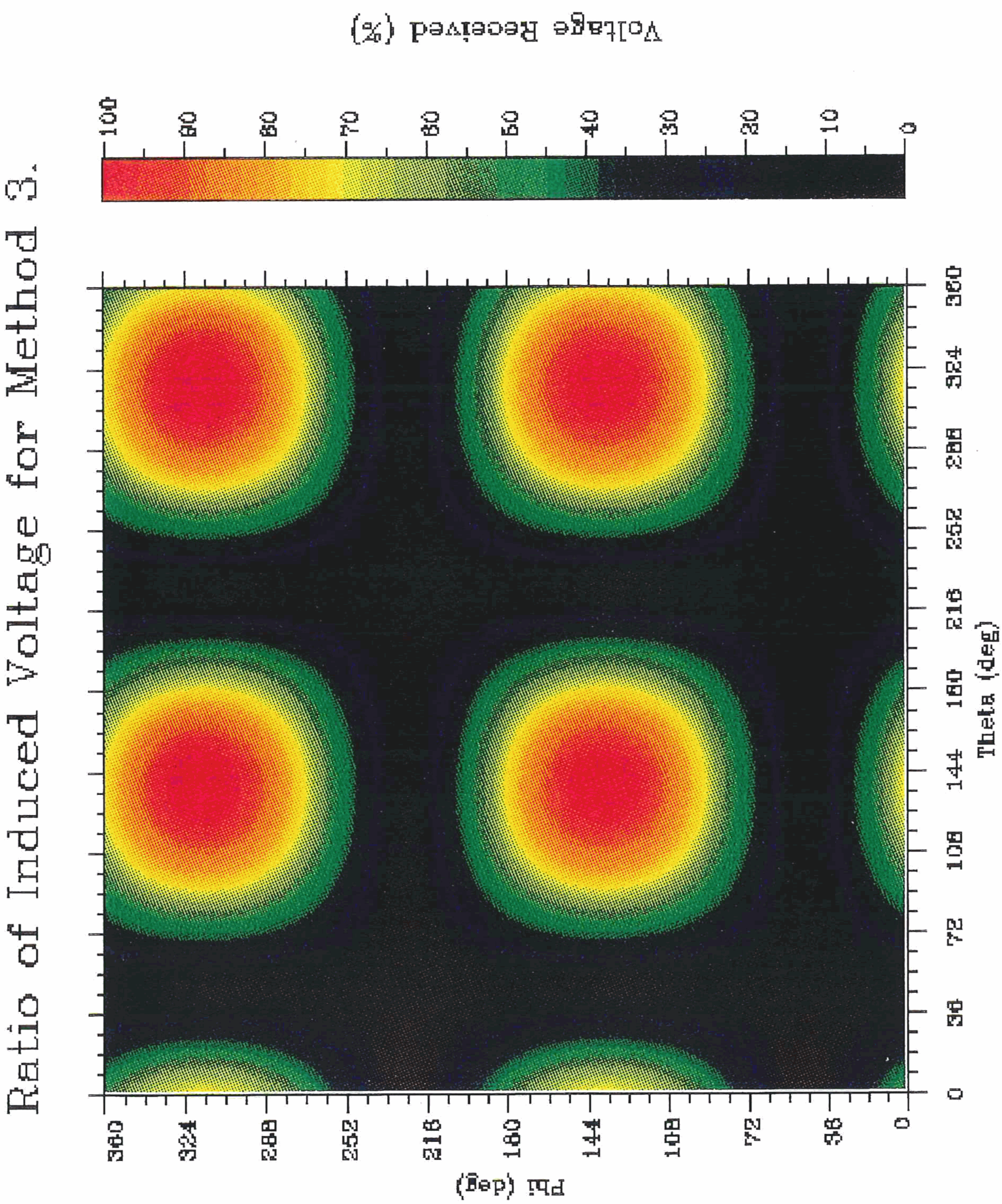
While the previous analysis investigated the dependence of the beacon's polarisation invarience upon β_B, similar reasoning can be applied to determine how closely the horizontally polarised receive antenna needs to be matched to the vertical array before horizontally polarised backscatter can be detected reliably. By extending equation (2.4b) to include this horizontally polarised receive array with a voltage gain ration β_R defined with respect to the existing vertical array, the expressions for received EMF and normalised power then become:



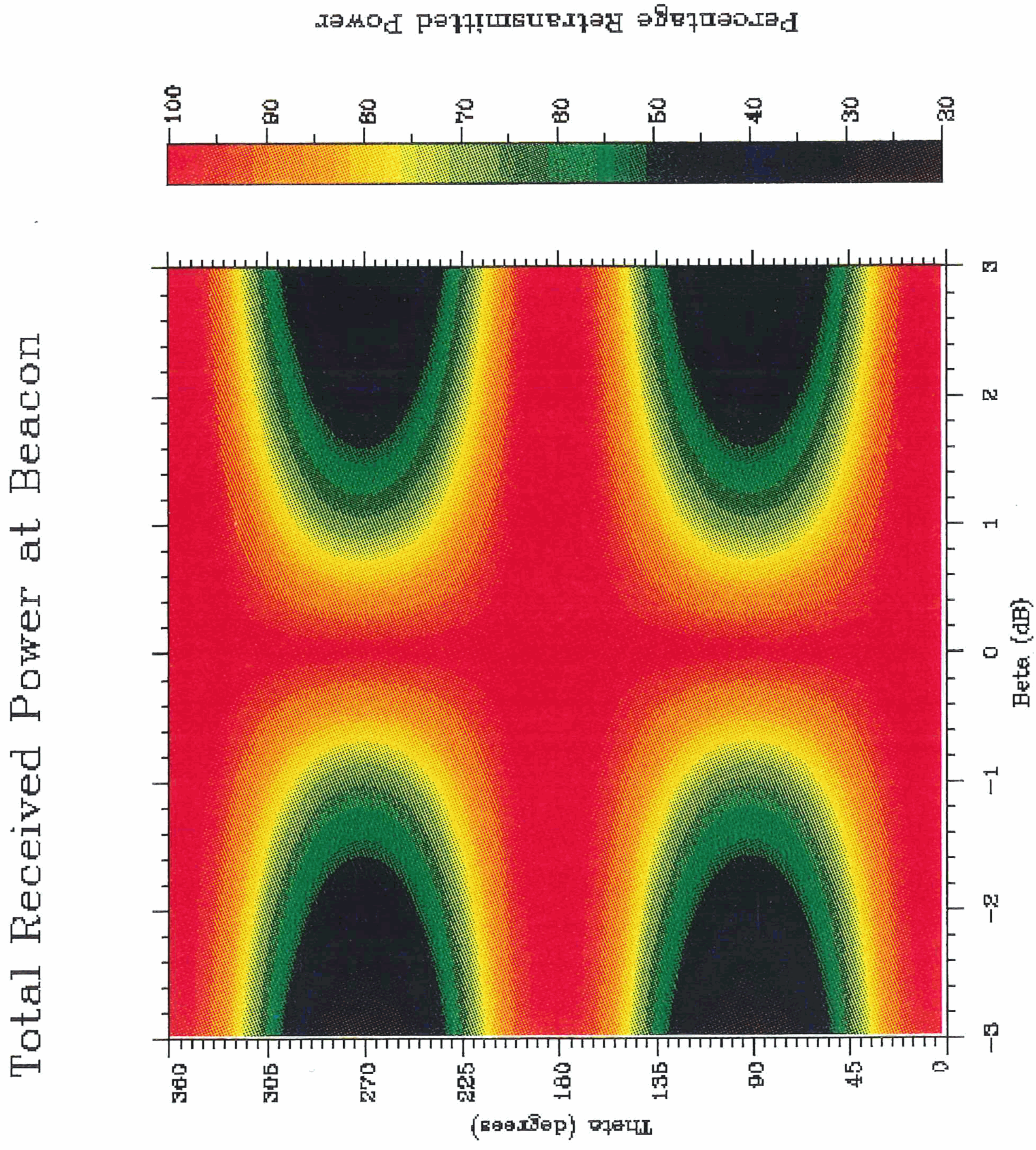
Graph 2.2 **Induced voltage ratios for beacon configuration 1.**



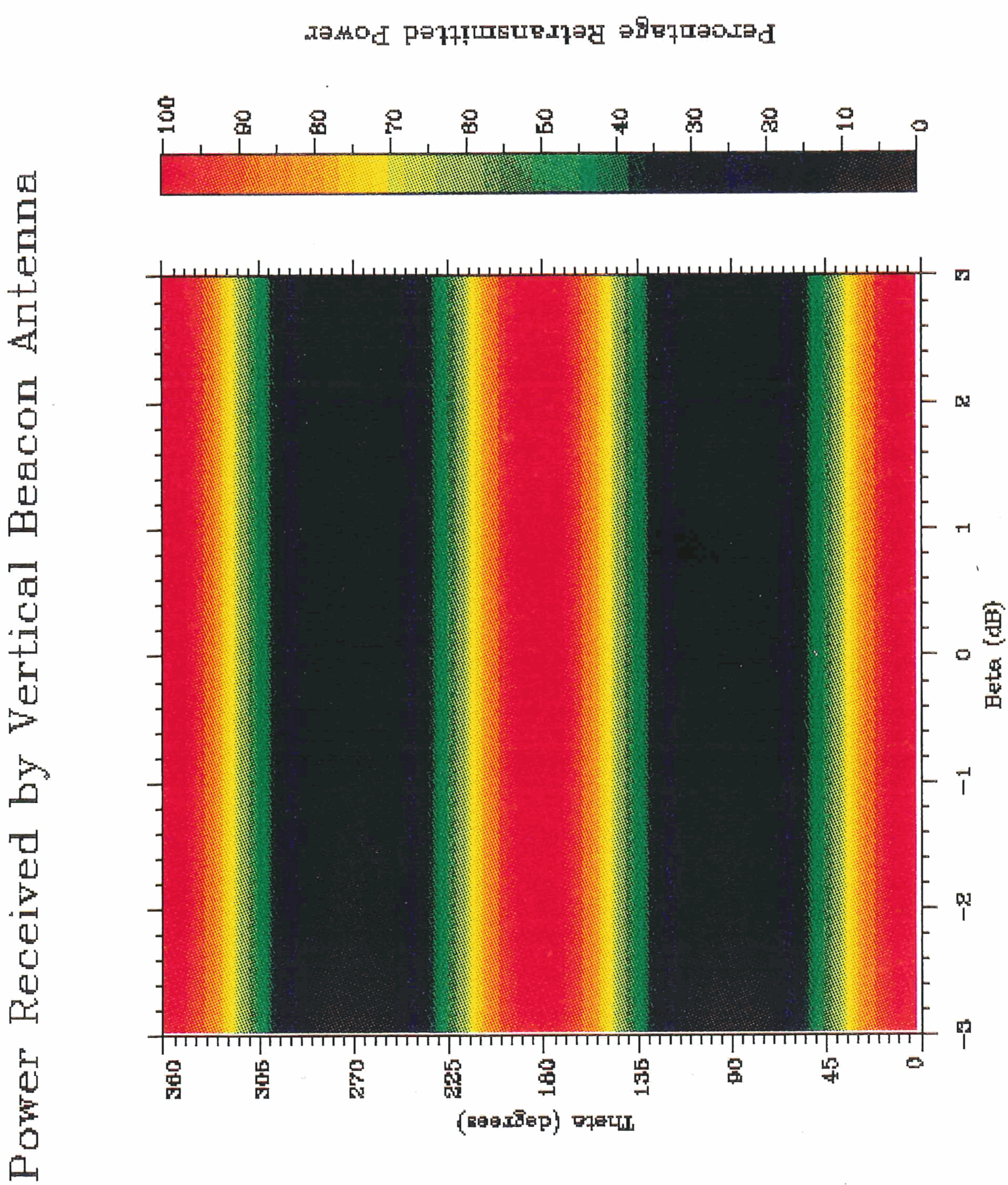
Graph 2.3 **Induced voltage ratios for beacon configuration 2.**



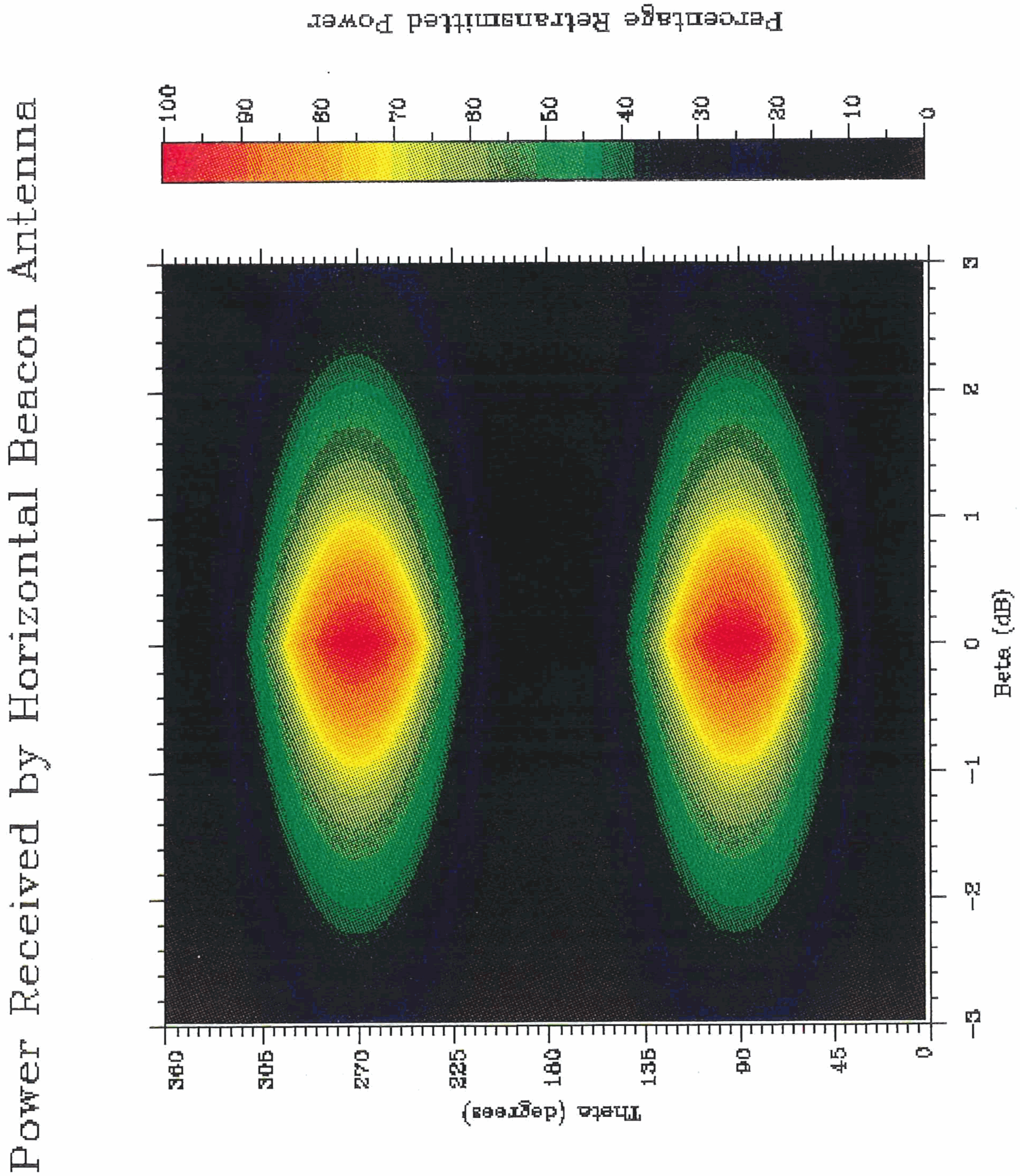
Graph 2.4 **Induced voltage ratios for beacon configuration 3.**



Graph 2.5 **Total received power as a function of β_R.**



Graph 2.6 **Vertical component of the received power as a function of β_R.**



Graph 2.7 **Horizontal component of the received power as a function of β_R.**

$$E_{R,Vertical} = E\left[\cos\theta\cos\phi + \beta_B^2 \sin\theta\sin\phi\right] \qquad (2.6a)$$

$$E_{R,Horizontal} = \beta_R E\left[\beta_B{}^2 \sin\theta\cos\phi - \cos\theta\sin\phi\right] \qquad (2.6b)$$

$$P_R = E^2\left[\cos^2\theta(\cos^2\phi - \beta_R^2\sin^2\phi) + \beta_B^2\sin^2\theta(\sin^2\phi + \beta_B^2\beta_R^2\cos^2\phi)\right] \qquad (2.6c)$$

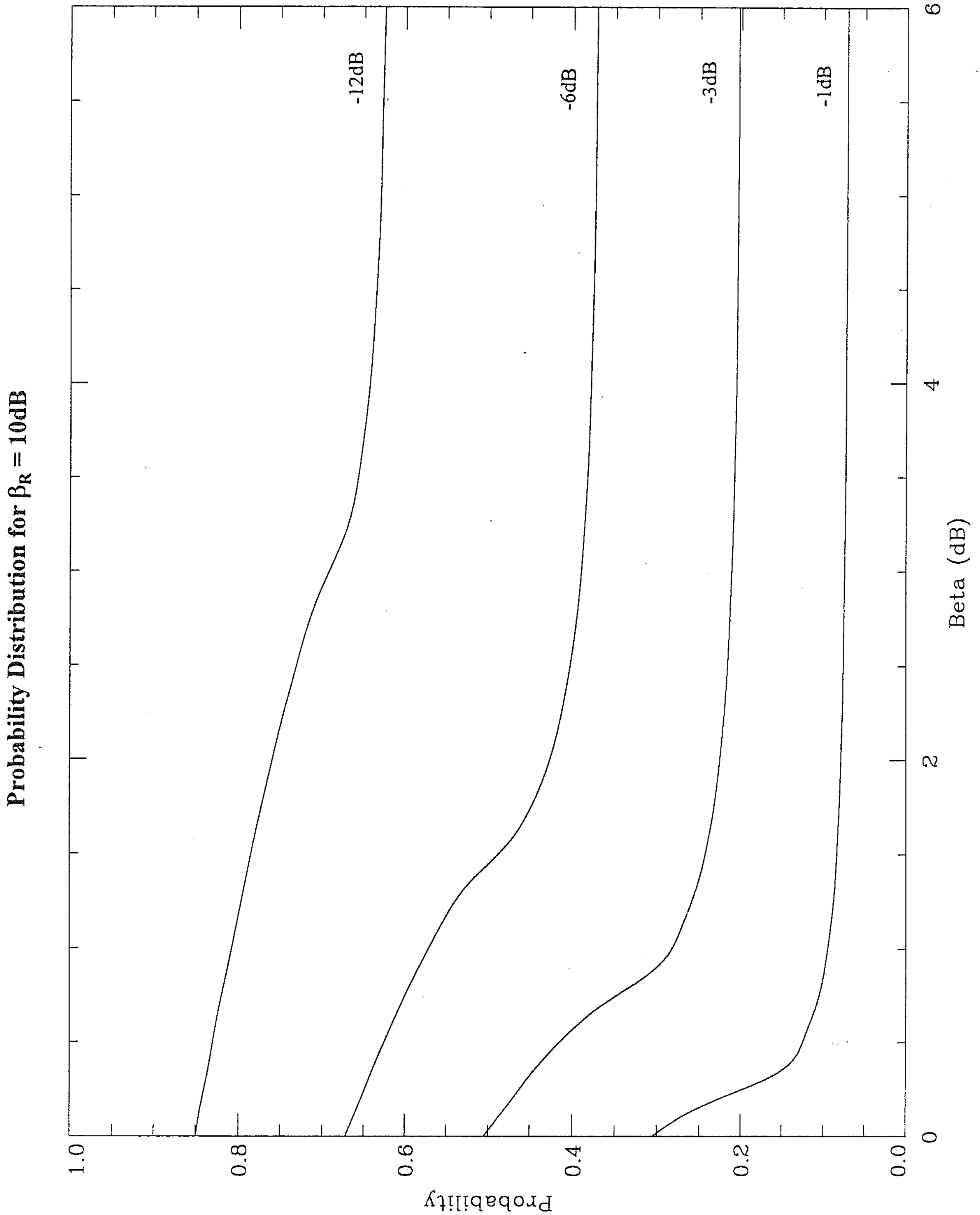
The probability of detecting target backscatter at specific levels below peak signal intensity was calculated by finding the ratio of the number of (ϕ,θ) values which gave a received signal greater than the desired level to the total pairs of (ϕ,θ) for specific values of β_B. Probability distributions as β_B varies from 0 to 6dB are shown in Graphs 2.8 and 2.9 where β_R=10dB and β_R=1dB respectively.

It was found that the effects of the horizontal receive array were negligible while β_R was greater than around 1.5dB, but as its gain approached the vertical array's the invariant to incoming polarisation became significant. Thus to minimise the effects of polarisation fading at the receive site, it would be necessary to install a horizontally polarised antenna of the order of the existing vertical array.

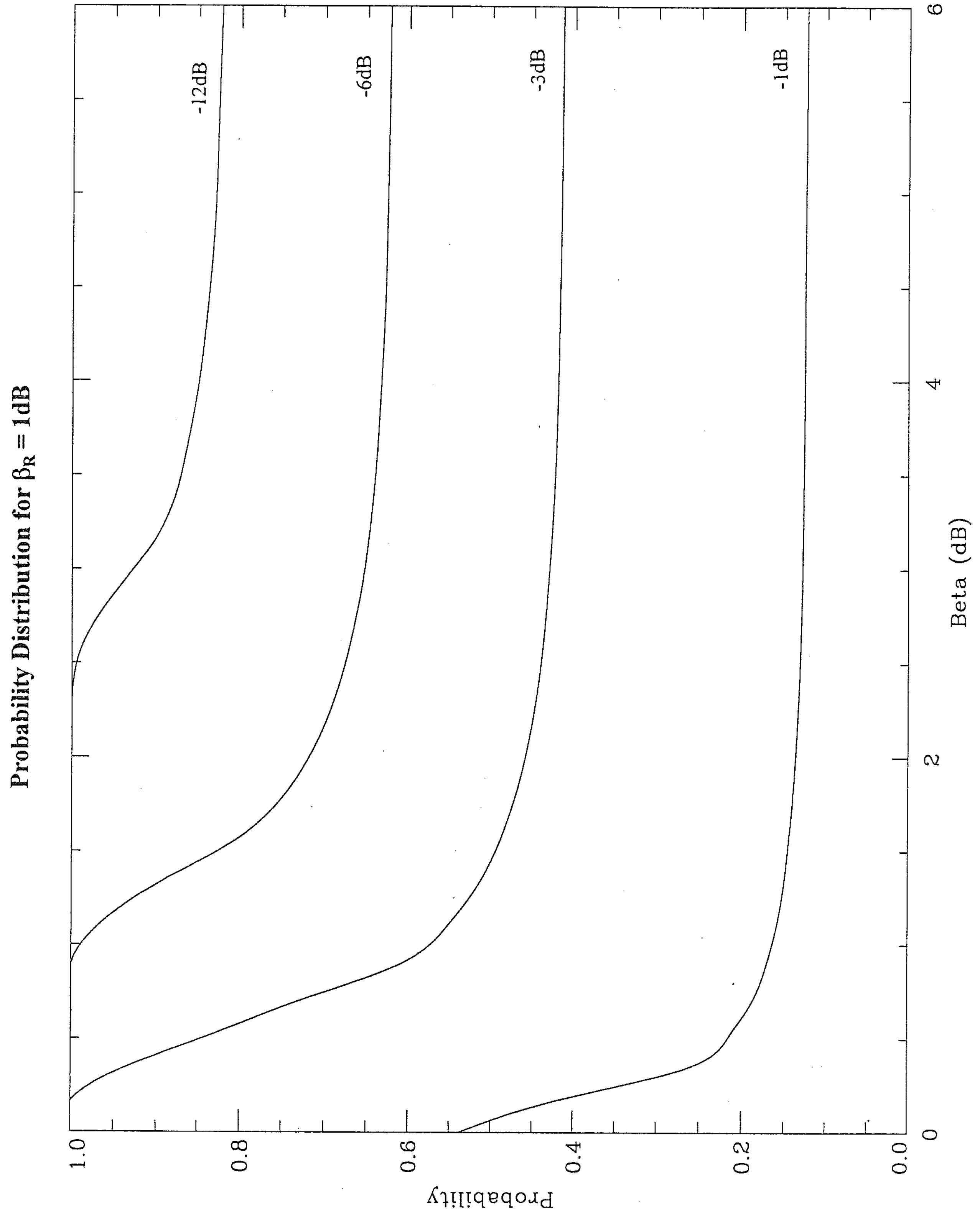
2.4 SUMMARY

In closing, it was found that for a system to be considered polarisation invariant, two orthogonal antennas are required, and their voltage gains need to be matched to within 1.5dB. While this is possible for a small structure such as a passive beacon, constructing a horizontally polarised receive array with a gain comparable to the existing vertical array was deemed infeasible, and other techniques to reduce the effects of polarisation fading are examined in the following section.

Also, it was found that while the absolute power loss due to the modulation of the beacon's load tended to 9.91dB as the voltage separation ratio tended to infinity, so long as the separation is maintained at above 20dB, the increase in losses can be kept below 1dB.



Graph 2.8 **Signal detection probability distributions for β_R=10dB.**



Graph 2.9 **Signal detection probability distributions for β_R=1dB.**

3. EFFECTS OF POLARISATION ROTATION

As OTH HF radar systems involve the passage of radio waves through the ionosphere they are vulnerable to the same phenomena that can adversely affect HF communications, including polarisation fading. It should be noted however, that unlike most HF communication systems, polarisation fading is only significant when the incoming signals' ordinary and extraordinary-rays cannot be resolved in group delay. While Faraday rotation provides a good approximation to the polarisation transformation of the ray as it propagates through the ionosphere, there are other causes of polarisation rotation which will not be discussed here.

Polarisation rotation occurs while the ray is in the ionosphere and results in a net change in polarisation angle at the receive site due to the different phase paths of the O and X-rays (for further detail see [3]). Thus under certain circumstances a target may be located in a region whose backscatter, as incident at the receive site, arrives horizontally polarised and is thus undetectable by the radar. As the construction of a large horizontally polarised receive array whose gain is within 1.5dB of the vertical array was determined to be impractical, the use of channel separation and spatial diversity were investigated as methods of minimising the effects of polarisation fading.

While it is desirable to keep the incident ray's polarisation parallel to the receive array, as the received power loss as a function of polarisation angle is given by:

$$P_{dB} = 20\log|\cos\theta| \tag{3.1}$$

it can be seen from Graph 3.1. that it is not necessary to keep the incident ray vertically polarised, and any system that can restrict θ to within $\pm 60^{\circ}$ of vertical, will constrain polarisation fading losses to less than 6dB.

3.1 ANALYTIC MODELS

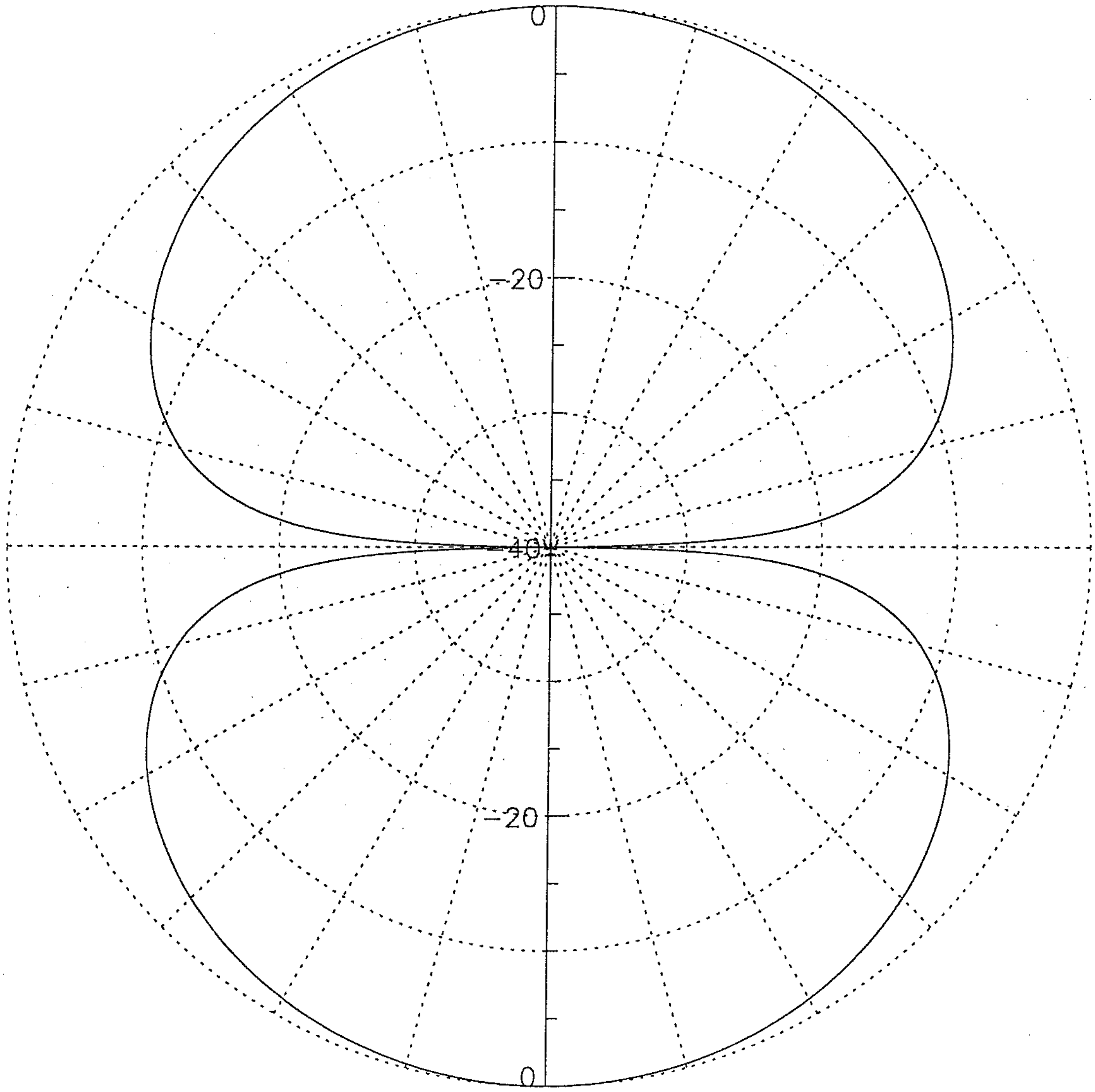
With the exception of the ray traced results, all calculations described hereafter are based upon the exact solutions to the Quasi-Parabolic Ionosphere model described in Croft[2], and assume low-angle propagation for maximum signal strength. As the QP model assumes a homogenous ionosphere, care should be taken to ensure relationships found to be valid in the static model still apply to a dynamic ionospheric model containing TIDs and other ionospheric perturbations.

Values of the ionospheric and physical constants used in all calculations are given in Appendix B, and the equations describing the ray parameters as derived by Croft are reproduced below:

$$D = 2r_0\left\{(\gamma - \beta_0) - \frac{r_0\cos\beta_0}{2\sqrt{C}}\ln\frac{B^2 - 4AC}{4C\left(\sin\gamma + \frac{1}{r_b}\sqrt{C} + \frac{1}{2\sqrt{C}}B\right)^2}\right\} \tag{3.2a}$$

$$P' = 2\left\{r_b\sin\gamma - r_0\sin\beta_0 + \frac{1}{A}\left[-r_b\sin\gamma - \frac{B}{4\sqrt{A}}\ln\frac{B^2 - 4AC}{\left(2Ar_b + B + 2r_b\sqrt{A}\sin\gamma\right)^2}\right]\right\} \tag{3.2b}$$

$$P = 2\left\{-r_0\sin\beta_0 + \frac{B}{4}\left[\frac{1}{\sqrt{A}}\ln\frac{B^2 - 4AC}{4\left(Ar_b + \frac{B}{2} + \sqrt{A}r_b\sin\gamma\right)^2} + \frac{r_m}{\sqrt{C}}\ln\frac{B^2 - 4AC}{4C\left(\sin\gamma + \frac{1}{r_b}\sqrt{C} + \frac{1}{2\sqrt{C}}B\right)^2}\right]\right\} \tag{3.2c}$$



Graph 3.1 **Power loss as a function of polarisation angle.**

where

$$F = \frac{f}{f_c}$$

$$A = 1 - \frac{1}{F^2} + \left(\frac{r_b}{Fy_m}\right)^2 \tag{3.3a}$$

$$B = -\frac{2r_m r_b^2}{F^2 y_m^2} \tag{3.3b}$$

$$C = \left(\frac{r_b r_m}{Fy_m}\right)^2 - r_0^2 \cos^2 \beta_0 \tag{3.3c}$$

These equations were derived for the no-field case, so an effective frequency shift based upon the Quasi-Longitudinal approximation was used to determine the ray parameters for the ordinary and extraordinary waves. Given these values, the total polarisation rotation between two end points can be calculated as:

$$\varphi = \frac{1}{2}\left(\varphi^{+} - \varphi^{-}\right) \tag{3.4a}$$

$$\because \varphi^{\pm} = \frac{2\pi f}{c} P^{\pm} \tag{3.4b}$$

where P^{+} is the phase path of the ordinary wave and P^{-} the phase path of the extraordinary wave evaluated for the desired ground range and the frequencies:

$$f^{\pm} = f_0 \pm f_{gm} \tag{3.5}$$

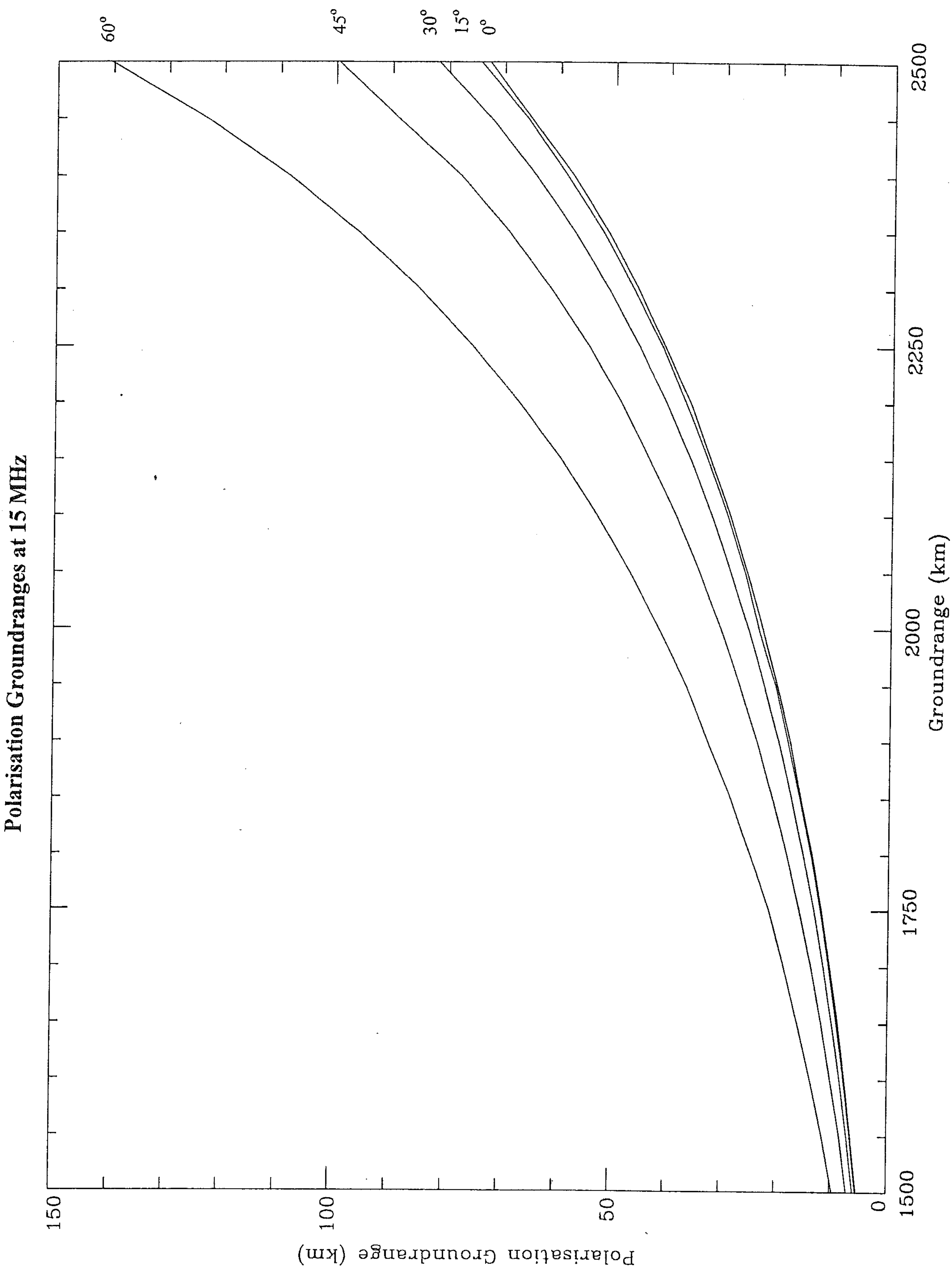
The initial elevation angles $\beta^{\pm}_0$ were evaluated for a given ground range D_0, and an operating frequency f_0, by solving equation (3.2a) numerically and substituting this value into (3.2b) and (3.2c).

3.2 SPATIAL DIVERSITY

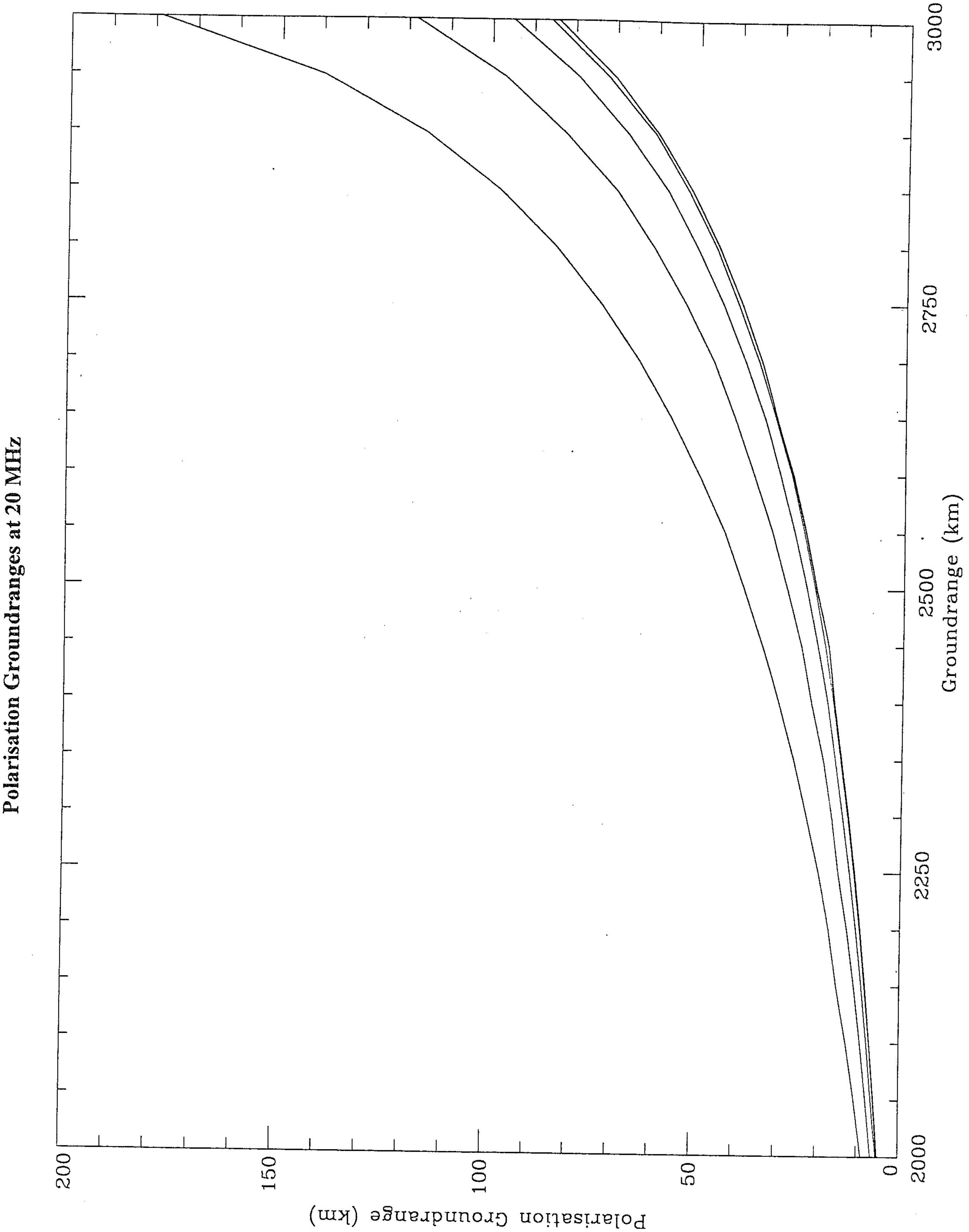
The change in polarisation angle with ground range was investigated to determine what variation in ground range will cause the polarisation to go from horizontal to vertical for targets located within the radar's footprint.

The required change in ground range (defined here to be the polarisation ground range) was evaluated by finding numerically what ΔD corresponds to a $\Delta\varphi$ of 90°. Using this technique the polarisation ground range was found to be in the tens to hundreds of kilometres and azimuthal variations for operational frequencies of 15MHz and 20MHz are shown in Graphs 3.2 and 3.3.

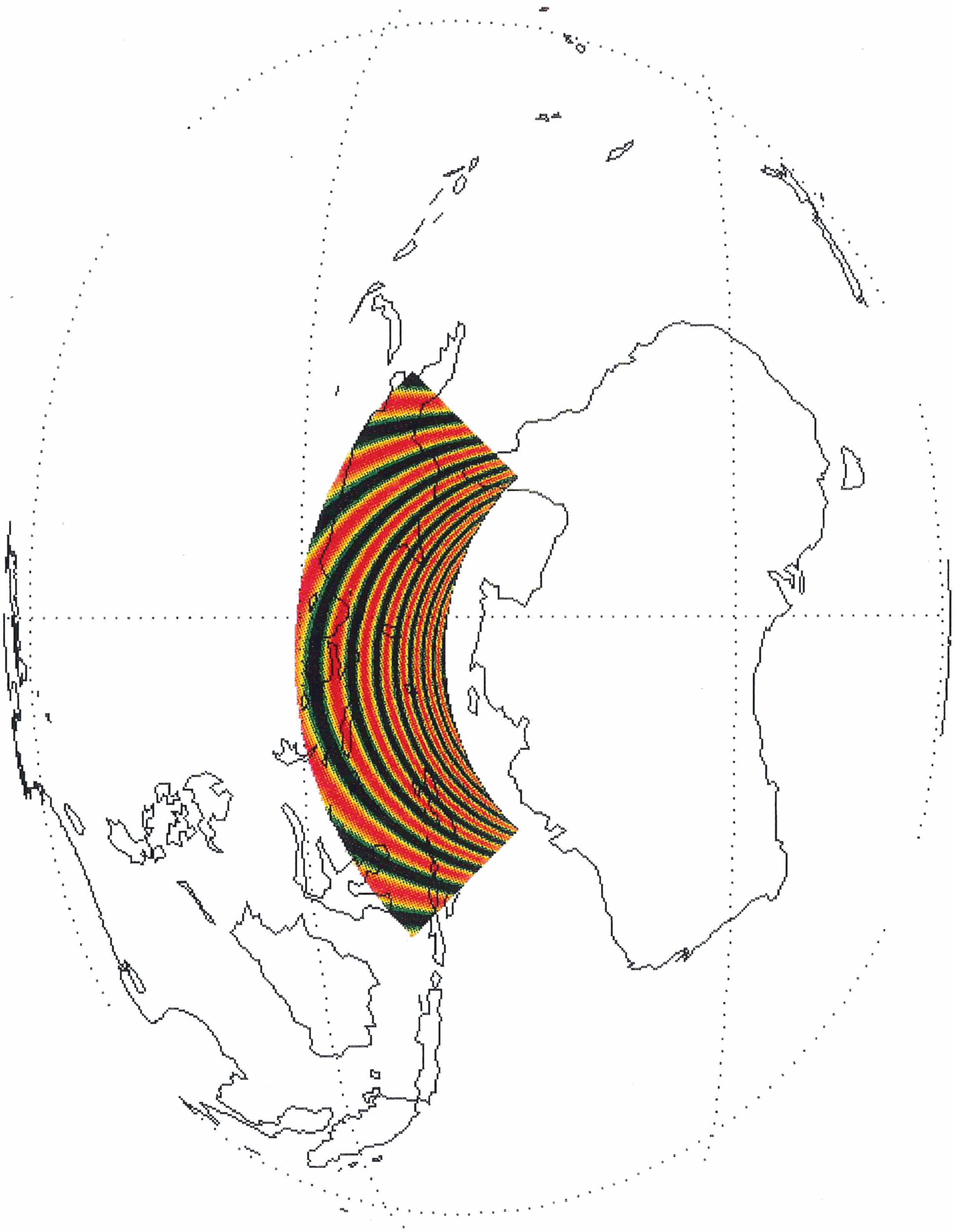
Thus given these ground ranges, one can see that even for an aircraft travelling at approximately 1000 kph, it would take between half a minute and ten minutes for the target's backscatter to change from being horizontally to vertically polarised. Given this slow rate of change of polarisation, any method that relies solely upon spatial diversity would require many dwells before a target hidden in a null would become visible, even if a rotation of only 30 degrees is required. The regions of illumination when the radar is operating at 80% of the MUF are shown in Graph 3.4. It should be noted that while these distances were calculated using the Quasi-Parabolic model, the introduction of even small-scale TIDs could cause



Graph 3.2 **Polarisation Ground ranges for 15 MHz.**



Graph 3.3 **Polarisation Ground ranges for 20 MHz.**



Graph 3.4 Geographic distribution of polarisation fringes.

significant deviations from the predicted ground ranges due to path differences associated with a non-stratified ionospheric model.

3.3 CHANNEL SEPARATION

The use of channel separation takes advantage of the dependence of polarisation upon the operating frequency. Thus, to achieve a desired change in the polarisation of the backscatter from a specific ground range, a corresponding frequency shift could be introduced.

In this report, two possible ways that channel separation could be used to reduce polarisation fading in OTHRs are examined; one involves operating the radar at two alternating frequencies such that the backscatter's polarisation changes by 90°, the other would involve measuring the polarisation of the backscatter from the desired ground range, and changing the operating frequency such that the backscatter incident at the receive site is now vertically polarised.

While the former is simpler to implement and effectively allows the reception of both horizontally and vertically polarised rays, because of the time multiplexing, each virtual array is useable only 50% of the time, In contrast, the frequency correction method would allow for optimum target detection by ensuring the received backscatter remains approximately vertically polarised. Alternatively, the real time polarisation correction could also be used to ensure that the transmitted signal is horizontally polarised when incident upon oceanic regions, thus maximising the signal to clutter ratio [5]. To do this, an approximation to $\frac{\partial\varphi}{\partial f}$ could be used to calculate the required frequency jump analytically.

One obvious problem in implementing this frequency-shift is that ionospheric perturbations such as fluctuations in electron density can cause predicted frequency changes to quickly become inaccurate, while a non-stratified layer can cause significant path deviations due to the aperiodic modulation of the refractive index along the ray's path of propagation. Thus any method that utilises frequency correction would need to taken such variations into account.

To investigate the magnitude of the frequency shift required to cause a 90° rotation (polarisation bandwidth), equation (3.4a) was solved numerically to find what Δf corresponded to a $\Delta\varphi$ of 90° at any given operating frequency. Below are specific polarisation bandwidths as a function of azimuth, with additional values shown graphically in Graph 3.5 to 3.7.

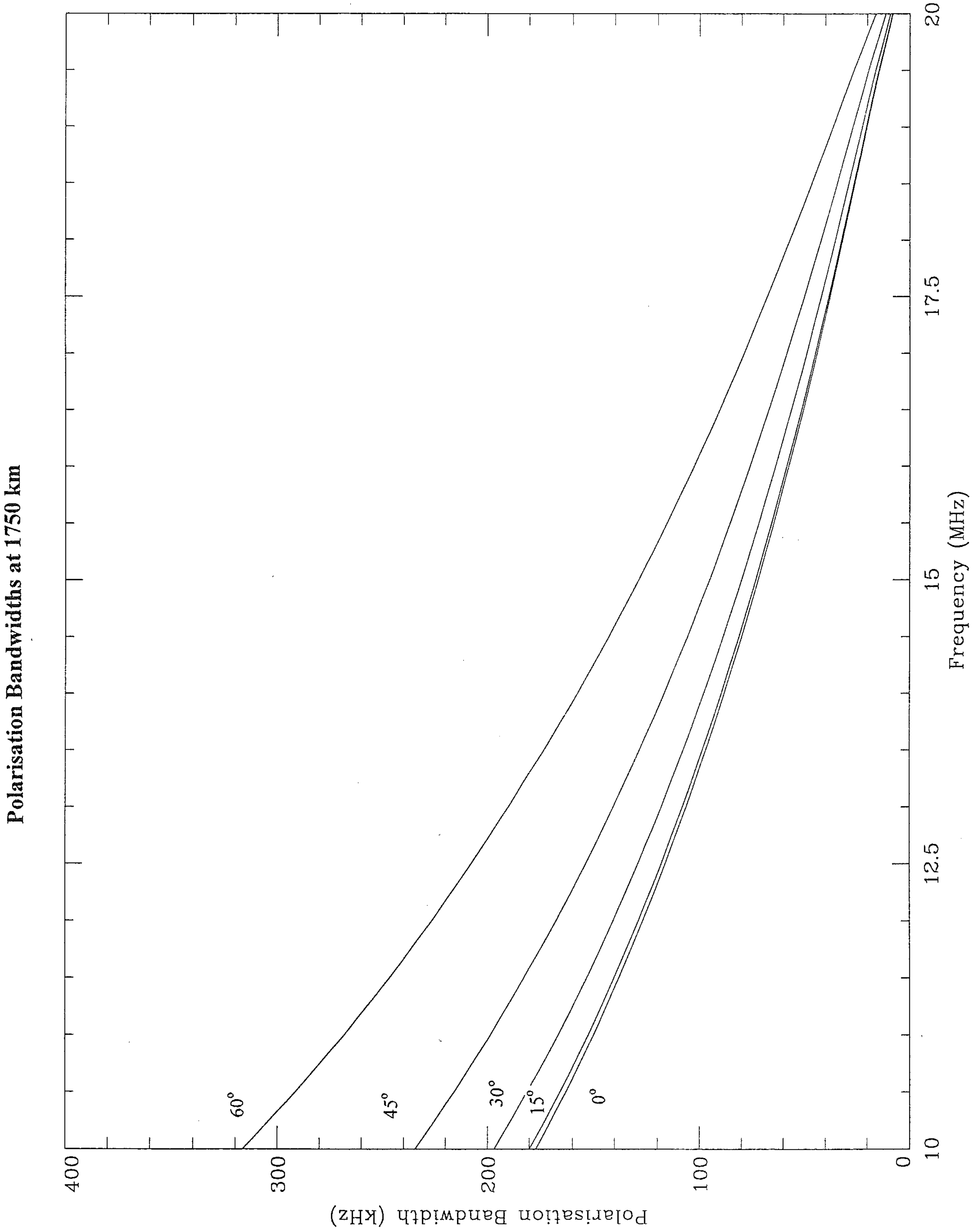
Operating Frequency (MHz)	Polarisation Bandwidth (kHz)			
	0° E of N	15° E of N	30° E of N	45° E of N
10	220.9	224.6	251.3	290.0
12	164.1	167.0	181.9	216.3
14	120.4	122.3	133.3	158.7
16	84.9	86.8	94.9	112.4
18	56.0	57.1	62.1	74.6
20	30.6	31.1	34.3	40.9

Table 3.1 Polarisation Bandwidths for selected frequencies and azimuths for a 2000km ray path.

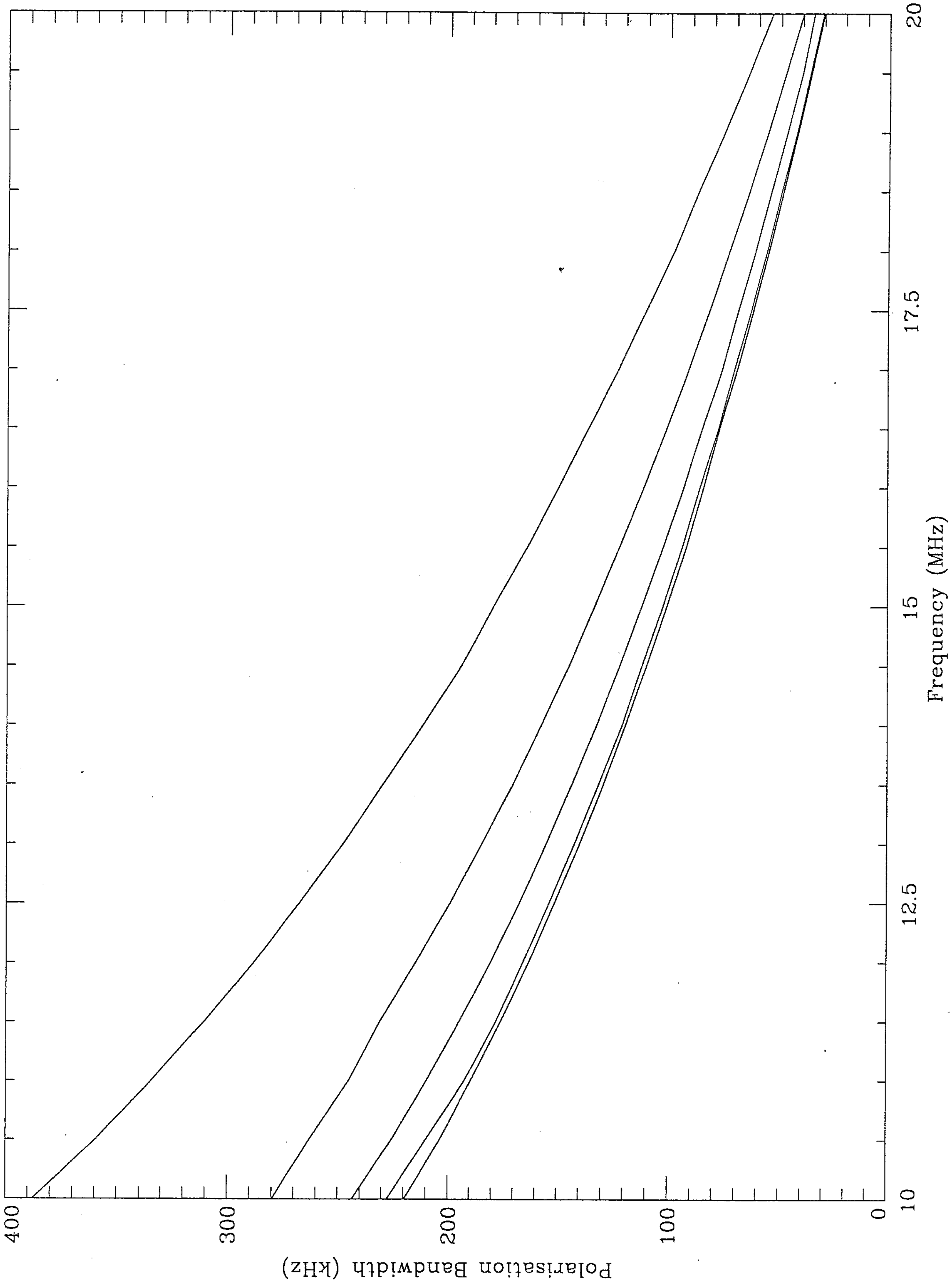
These values agree with the calculations described in [4]. Given a fixed ground range, it was found that for changes in frequency of the order of the polarisation bandwidth, $\frac{\partial\varphi}{\partial f}$ is almost constant. Thus, one way to implement the frequency correction method would involve using a linearised approximation to the gradient which gives:

$$\left.\frac{\Delta f}{\Delta\varphi}\right|_{f_0} = \frac{c}{\pi\left[(P'^{+} - P^{+}) - (P'^{-} - P^{-})\right]} \tag{3.6a}$$

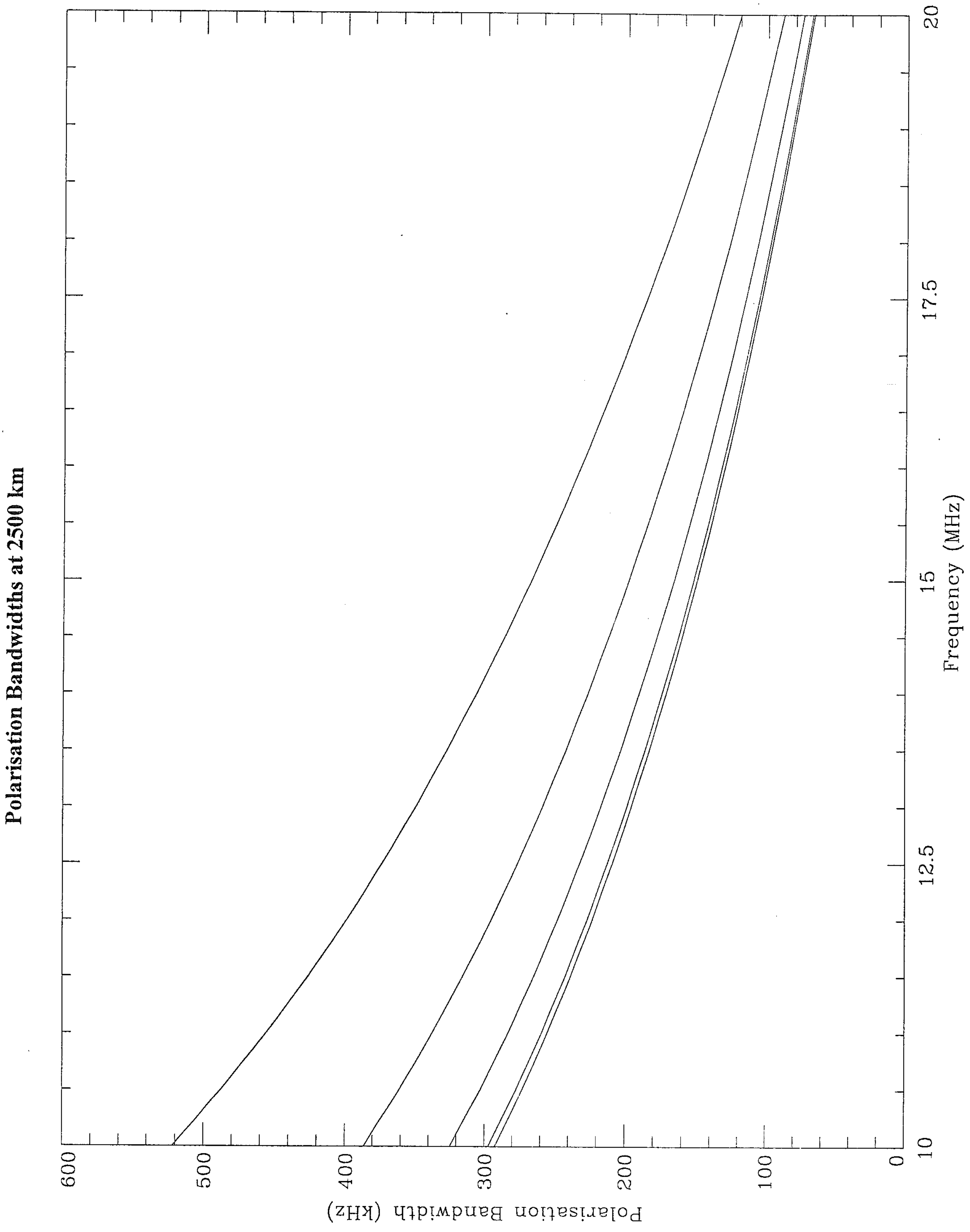
$$\because \left.\frac{\partial\varphi}{\partial f}\right|_{f_0} = \frac{\pi}{c}\left[(P'^{+} - P^{+}) - (P'^{-} - P^{-})\right] \tag{3.6b}$$



Graph 3.5 **Polarisation Bandwidths for a ground range of 1750km.**



Graph 3.6 **Polarisation Bandwidths for a ground range of 2000km.**



Graph 3.7 **Polarisation Bandwidths for a ground range of 2500km.**

where

$$\frac{\partial\varphi^{\pm}}{\partial f} = \frac{\partial\varphi^{\pm}}{\partial P}\frac{\partial P}{\partial f}$$
$$= \frac{2\pi f}{c} \times \frac{1}{f}\left(P'^{\pm} - P^{\pm}\right) \qquad (3.7)$$

It was found that the approximation (3.6a) accurately predicts the change in frequency required to rotate the polarisation vector to within a few degrees of $\Delta\varphi$, as shown in Graph 3.8, where the error is the difference between the actual $\Delta\varphi$ and the calculated 90° rotation. One operational consequence is that under certain atmospheric conditions, it may be possible to estimate the required change in frequency based upon purely empirical data. Given this linear relationship, the gradient could then be evaluated by measuring the polarisation angle at two distinct operating frequencies, although temporal variations in the ionosphere's structure will affect the period of time for which this approximation remains valid.

3.4 COMPARISON WITH RAY TRACED VALUES

As stated earlier, all the results previously discussed were generated using the simple quasi-parabolic model of the ionosphere, and thus are subject to the limitations of the model. To investigate how reliable these relationships are when a dynamic ionospheric model is introduced, a raytracing package developed by Coleman[6] was used into which travelling ionospheric disturbances were introduced.

Under static conditions, the raytracing package agreed closely with the both the exact and approximate analytic results (see Table 3.2), but these calculations became less accurate as larger scale disturbances were introduced.

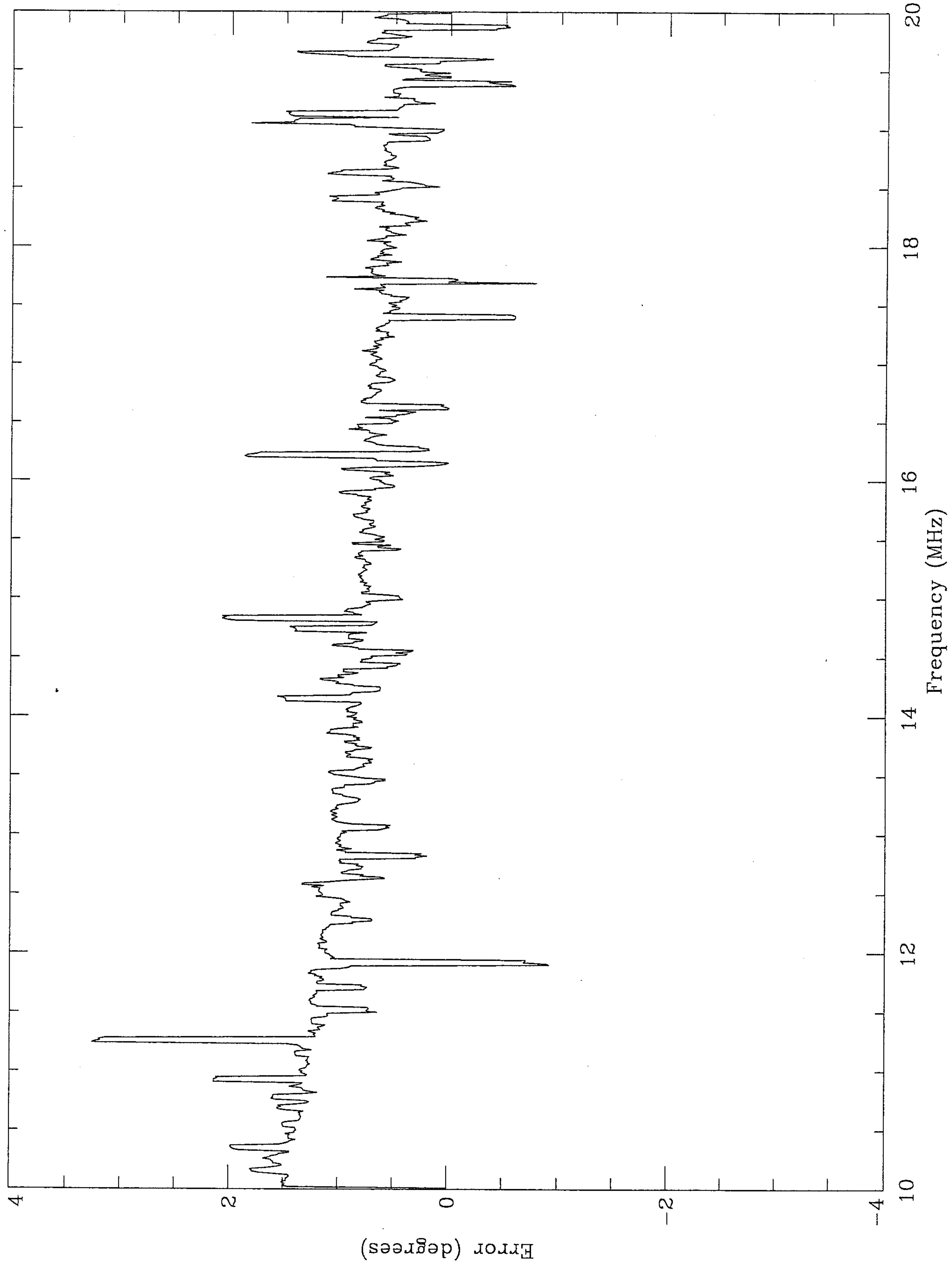
Operating Frequency (MHz)	Polarisation Bandwidth (kHz)		
	Croft's QP	Linearised Model	Ray Traced
10	220.9	224.7	230.9
12	164.1	166.1	161.9
14	120.4	121.6	118.9
16	84.9	85.9	78.6
18	56.0	56.3	52.2
20	30.6	31.0	30.7

Table 3.2 Comparison of Polarisation Bandwidths for a 2000km ray path. obtained using different models.

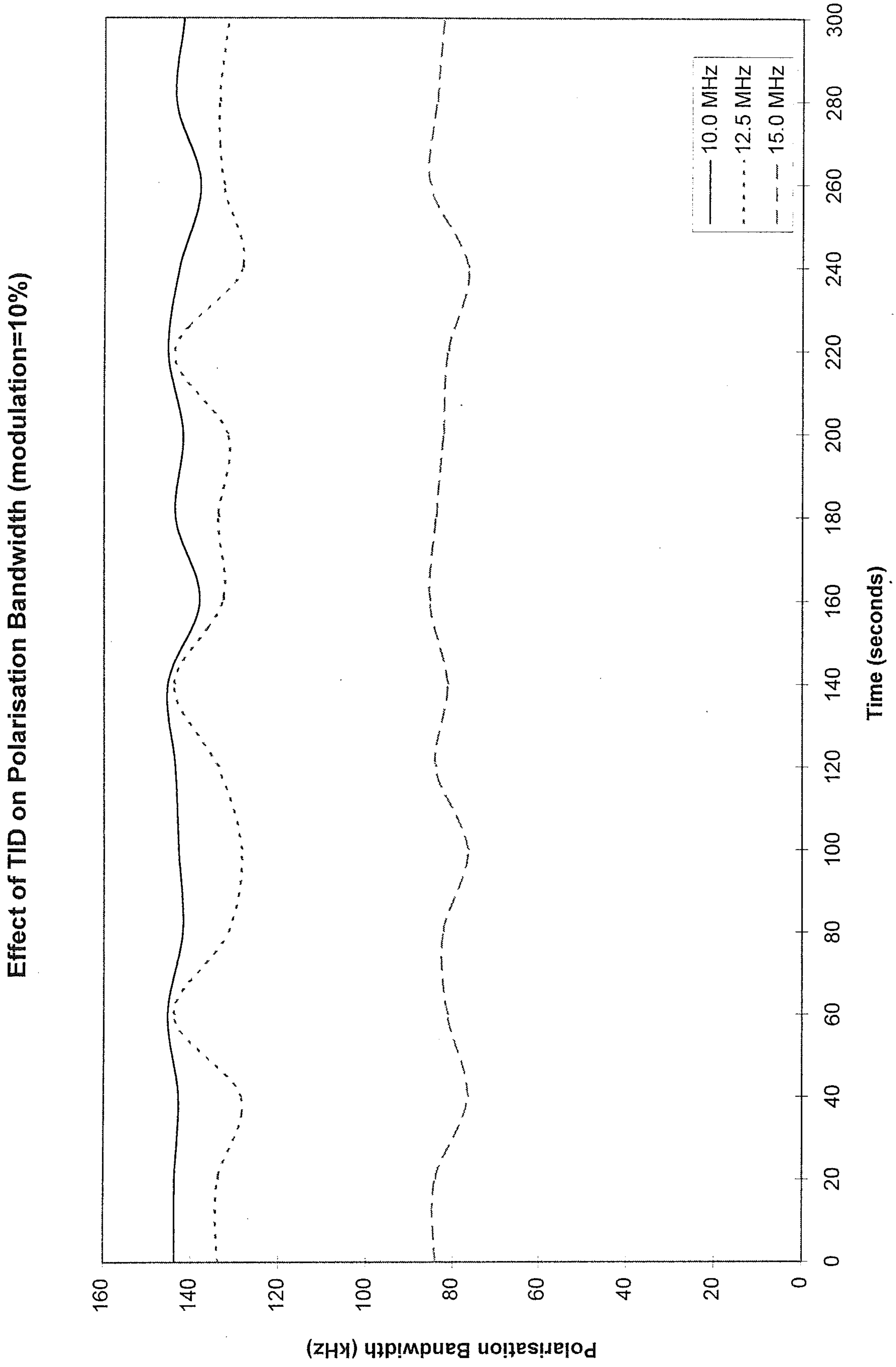
To determine how sensitive the linearised predictions are to changes in ionospheric parameters, a sinusoidal TID that modulated the maximum electron density was introduced into the system. As can be seen from Graphs 3.9 and 3.10, the introduction of the TID likewise results in a modulation of the polarisation bandwidth over time. When the TID's modulation was set to 10% variations of up to 20kHz were found, while at 40% the polarisation bandwidth deviated by as much as 40kHz from the homogeneous case.

Given the magnitude of these fluctuations, it is obvious that any method that uses $\frac{\partial\varphi}{\partial f}$ to predict a frequency jump can quickly become inaccurate once the maximum electron density, and thus the polarisation bandwidth varies by more than several percent. Using the initial gradient (ie. at t=0), frequency corrections were made when ever the incoming ray approached 60° off vertical, and as can be seen from Graph 3.11, even with a TID modulation of only 5%, the changing electron density can cause a prediction to be as far as 20° off, although even this error corresponds to less than 1dB of loss.

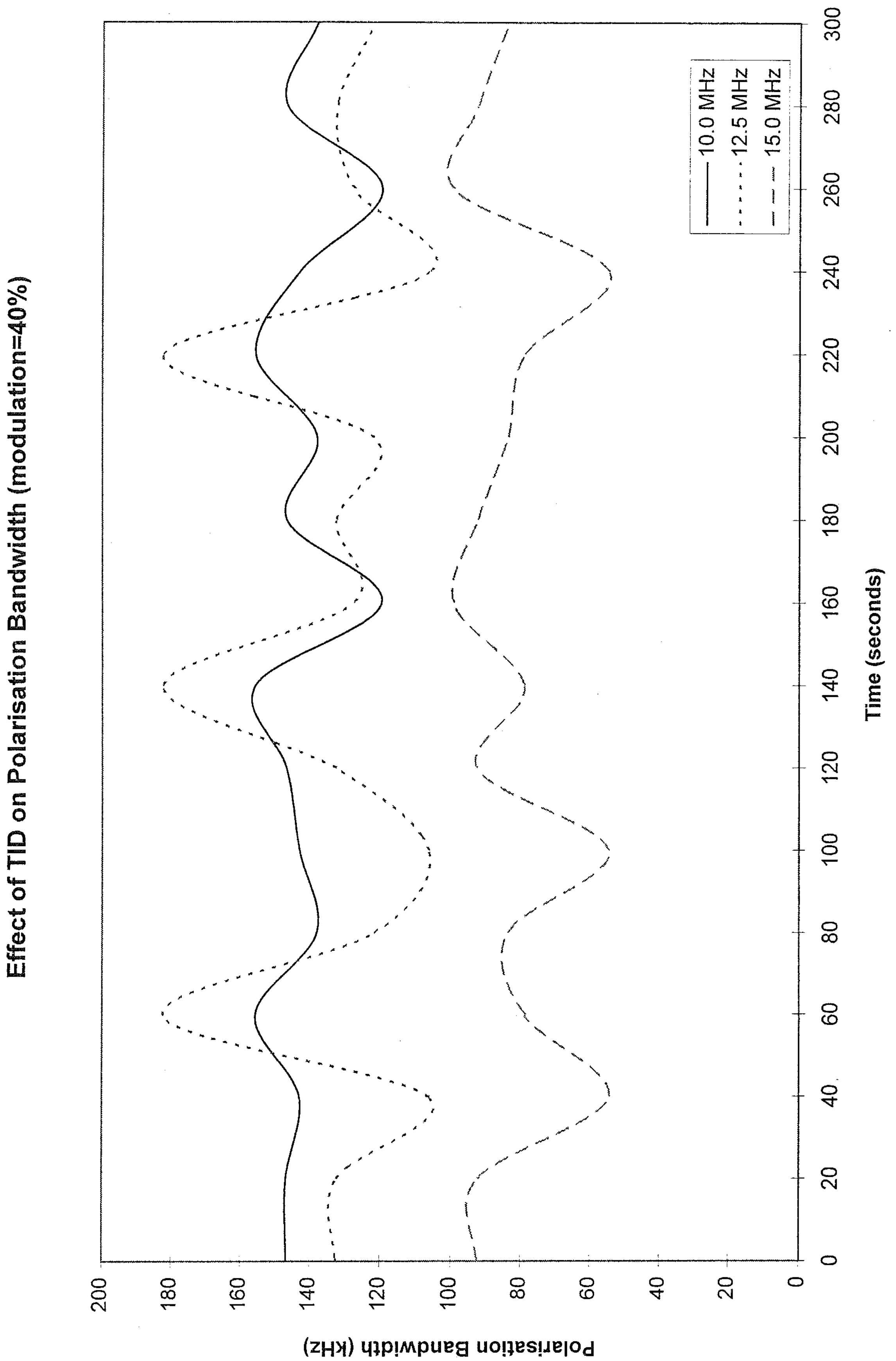
Thus, while the linear relationship can be used to accurately predict Δf given $\Delta\varphi$, any technique that tries to actively minimise polarisation losses will need to take into account the perturbations caused by ionospheric irregularities and TIDs.



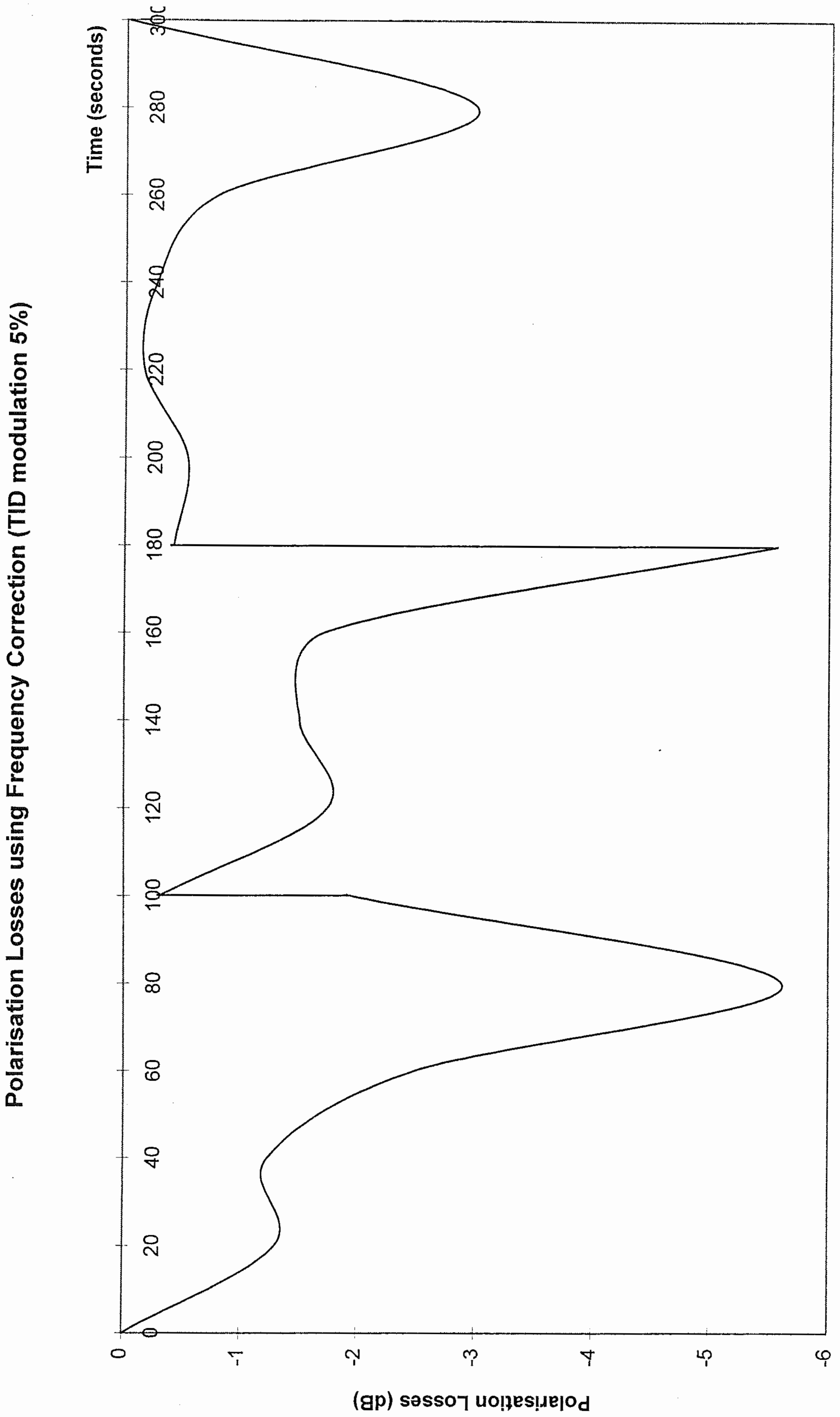
Graph 3.8 **Error in predicting a 90° correction using the linearised model.**



Graph 3.9 **TID modulation of Polarisation Bandwidths.**



Graph 3.10 **TID modulation of Polarisation Bandwidths.**



Graph 3.11 **Effect of frequency corrections on polarisation losses.**

4. OTHER CAUSES OF FADING

While the focus of this project has been the effects of polarisation fading on HF radar systems, it is by no means the only cause of signal fading; there are other phenomena that can cause significant fluctuations in the backscatter's amplitude, of which interference and diffraction fading are the most significant. Apart from being able to resolve reflections off different ionospheric layers and thus eliminate the multimodal fading that plagues other HF systems, OTH radars are also relatively immune to absorption and other forms of fading whose period is much greater than the dwell time. Shown below are the most common types of HF signal fading, and their approximate periods [7, 8]:

Fading Type	Cause	Fading Period	Relative fading depths
Flutter	Small irregularities in the F-layer	10-100 ms	medium
Diffraction	Movement of ionospheric irregularities	5-20 s	medium
Polarisation	Rotation of polarisation ellipse	10-100 s	large
Interference (focusing)	Changes in curvature of the reflecting layer	15-30 min	large
Absorption	Time variation of ionospheric absorption	60 min	small

Table 4.1 Summary of HF fading phenomena

Flutter is primarily caused by the high speed convection of irregularities in the F-region, where velocities above $100ms^{-1}$ are not uncommon. Due to its short fading period, the effects of flutter should be visible in the raw received radar signal. Flutter has been found to have a fading correlation bandwidth of greater than 1kHz, and its distortion of AM signals is more severe than other modulated signals, due to the destructive interference caused by multiple rays being reflected off these F-layer irregularities. It is most common at times of spread-F, near sunset and especially near the equinoxes.

Diffraction fading occurs when a strong ionospheric irregularity causes destructive interference between waves that have diffracted around it. This phenomena, which usually lasts for less than twenty seconds but can persist for up to several minutes, can be modelled in terms of diffraction around a cylindrical non-absorbing irregularity. This form of fading is also frequency dependent, with the fading depths increasing proportionally with wavelength.

Interference fading is one of the most common forms of fading and is caused by interactions between multiple waves reflected off an irregular ionospheric layer. This phenomena is also described as focusing/defocusing and may occur on both oblique and vertical propagation paths, although its effects become less significant with increasing frequency. These perturbations in ionospheric reflectors (ie. any surface of constant electron density) have several causes including the passage of acoustic-gravity waves through the neutral ionosphere and large scale TIDs (up to thousands of kilometres long) travelling at speeds between $100ms^{-1}$ and $1000ms^{-1}$.

As HF waves experience most of their signal attenuation while travelling through the D-layer, absorption fading is caused by changes in the structure of the D-region. Although it only exists during day-time, the movement of irregularities in the D region, changes in ionising solar radiation and monthly variations in absorption can causes noticeable fade outs. As these periods can last anywhere between minutes to around an hour, it is only discernible in OTH radars by long term variations in backscatter and clutter signal strengths, although the SCV should remain relatively unchanged.

5. REFERENCES

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[6] Coleman, Dr C. (private communication), High Frequency Radar Division, Defence Science and Technology Organisation Salisbury

[7] Maslin, N. (1987), HF Communications: A System Approach (Pitman Publishing, London), p77

[8] Davies, K. (1989), Ionospheric Radio (Peter Peregrinus Ltd, UK), chapters 7

APPENDIX A: Derivations and Proofs

$$E_{B,V} = E\cos\theta \qquad E_{B,H} = \beta_{BE} E\sin\theta$$
$$E_{R,V} = E\cos\theta\cos\phi \qquad E_{R,H} = -\beta_B^2 E\sin\theta\cos(\phi + \tfrac{\pi}{2}) = \beta_B^2 E\sin\theta\sin\phi$$

$$\Rightarrow E_R = E(\cos\theta\cos\phi + \beta_B^2\sin\theta\sin\phi)$$

Equation A.1 Induced EMF at the receive site using beacon method 2.

$$E_{B,V} = \tfrac{E}{2}[\cos\theta - \beta\sin\theta] \qquad E_{B,H} = \tfrac{\beta_B E}{2}[\beta_B\sin\theta - \cos\theta]$$
$$E_{R,V} = \tfrac{E}{2}[\cos\theta\cos\phi + \beta_B^2\sin\theta\sin\phi] \qquad E_{R,H} = \tfrac{\beta_B E\sin\phi}{2}[\beta_B\sin\theta - \cos\theta]$$

$$\Rightarrow E_R = \tfrac{E}{2}[\cos\theta\cos\phi - \beta_B\sin\theta\cos\phi + \beta_B^2\sin\theta\sin\phi - \beta_B\cos\theta\sin\phi]$$

Equation A.2 Induced EMF at the receive site using beacon method 3.

$$E_{B,V} = E\cos\theta \qquad E_{B,H} = \beta_B^2 E\sin\theta$$
$$E_{R,V} = E[\cos\theta\cos\phi + \beta_B^2\sin\theta\sin\phi] \qquad E_{R,H} = \beta_R E[\beta_B^2\sin\theta\cos\phi - \cos\theta\sin\phi]$$

$$\Rightarrow E_R^2 = E^2[\cos\theta(\cos\phi + \beta_R\sin\theta) + \beta_B^2\sin\theta(\sin\phi + \beta_R\cos\phi)]^2$$

Equation A.3 Induced EMF at the receive site after introducing the horizontal antenna array

Define $\beta_B \in [0,1]$, since we assume finite antenna gain.

Then, $P_R = \|E_R\|^2$

$$\frac{\partial P_R}{\partial \beta_B} = \frac{\partial}{\partial \beta_B}\|E_R\|^2$$
$$= 2\beta_B\|\sin\theta\sin\phi\| \times 2\|\cos\theta\cos\phi + \beta_B^2\sin\theta\sin\phi\|$$
$$= 0 \ \forall\theta,\phi \Leftrightarrow \beta_B = 0$$

but,

$$\frac{\partial^2 P_R}{\partial \beta_B^2} \geq 0$$

$\Rightarrow \beta_B = 1$ maximises received power.

Equation A.4 Proof for optimal beacon gain ratio

APPENDIX B: List of Symbols

Symbol	Description
θ	Polarisation rotation on outbound (transmitter-beacon) leg
ϕ	Polarisation rotation on inbound (beacon-receiver) leg
γ	Angle ray makes at the bottom of the ionosphere
φ	Polarisation (Faraday) rotation in radians
β_0	Initial elevation angle of ray
β_B	Voltage gain ratio (G_H:G_V) of beacon's antennas
β_R	Voltage gain ratio of receiver's antennas
D	Ground range (metres)
E	Amplitude of transmitted EMF
E_R	Instantaneous received EMF
N_M	Maximum electron density (10^{12} m^{-3})
P_R	Instantaneous receive power
P	Phase path (metres)
P'	Group path (metres)
r_0	Radius of the earth (6370km)
r_b	Radius to the layer base (r_m-y_m)
r_m	Radius at which electron density is at the maximum (300km)
y_m	Layer semithickness (100km)

APPENDIX C: Source Code and Explanation of Programs

Provided here is a brief description of the major programs used to generate the graphs and probability distributions detailed in this document followed by the source code itself:

RECEIVE_PLOT

Used to generate Graphs 2.1 to 2.4, and Graphs 2.8 and 2.9. Calculates received power as a function of ϕ and θ (polarisation angles on outgoing and incoming legs) and β_B for a user specified β_R, and then collates these signal powers to derive the probability distributions shown in Graphs 2.8 and 2.9. These are calculated by finding the ratio of the number (ϕ, θ) pairs whose return power is above a user specified level to the total number of points for a given β_B.

BEACON_PLOT

Used to generate Graphs 2.5 through 2.7. Displays the voltage induced at the beacon as a function of polarisation angle and antenna gain ratios. Calculates the data for $\beta_B = 0$ through to the user specified maximum (MAX), and duplicates it for $\beta_B =$ -MAX to $\beta_B = 0$.

POLARISATION

Used to generate Graphs 3.2 through 3.8. Implementation the equations described in chapter 3 to calculate the polarisation rotation over a given ground range, the polarisation ground ranges and bandwidths as well as their linearised approximations. Also included are some routines that were used to evaluate the linearised model's accuracy.

ROUTINES

These routines are required by all other programs, and include the functions to solve the initial elevation angles, effective frequency shifts as well as calculate ray parameters such as ground range, phase path and group paths. The routine is courtesy of Dr. Chris Coleman.

```
;author: Peter Dreisiger, Dec 1996 IE HFRD DSTO SALISBURY
;==========================================================================
;This programme shows how the reradiated beacon power varies with BETA (in dB)
;using an uncoupled horizontal-vertical antenna pair
;==========================================================================

FUNCTION Mirror, dat
  fin = findgen(720, 360)
  ind = 359 -indgen(360)
  fin(0:359, * ) = dat
  fin(360: * , * ) = dat(ind, * )
  fin = congrid(fin, 360, 360)
  RETURN, fin
END

PRO Plot_graph, dat, title
  erase
  x_beta_value = ['-3', '-2', '-1', '0', '1', '2', '3']
  y_theta_value = ['  0', ' 45', ' 90', '135', '180', '225', '270', '305', '360'
]
  a = [1, 2]

  plot, a, /NODATA, POS=[98, 98, 460, 460], /DEVICE, TICKLEN= -0.02, XTICKS=6, $
   YTICKS=8, XMINOR=5, YMINOR=9, XTICKNAME=x_beta_value, $
   YTICKNAME=y_theta_value, XTITLE='!6Beta (dB)', YTITLE='!6Theta (degrees)'
  max_val = max(dat, MIN=min_val)
  dat = ie_bytscl(dat, DMIN=min_val, DMAX=max_val, TOP=!colour.top)
  color_scale, /VERT, MINVAL=100 * min_val, INT=10, MAJ=ceil((max_val -min_val)
* 10) , $
   XPOS=500, YPOS=100, BAR_LENGTH=360, TITLE='Percentage Retransmitted ' + $
   'Power', MAX_USEABLE_COL=!colour.top
  xyouts, 30, 520, title, CHARSIZE=2, /device
  tv, dat, 100, 100, /DEVICE
END

PRO Beacon_plot
  window, TITLE='Passive Beacon Return Power Analysis', XSIZE=800, YSIZE=600
  load_colours, "FMS Rainbow"
  beta = 6.0
  read, beta, PROMPT='Enter the maximum absolute value for Beta (db): '
  beta_max = exp( -alog(10) /20 * beta)

  theta_arr = findgen(360) /57.3
  beta_arr = beta_max +(findgen(360) /360 *(1 -beta_max) )
  identity = fltarr(360) +1
  beta_arr = beta_arr^2
  pby = cos(theta_arr)^2
  pby = mirror(pby##identity)
  pbx = sin(theta_arr)^2
  pbx = mirror(pbx##beta_arr)
  tot = pbx +pby

  plot_graph, pbx, '!6Power Received by Horizontal Beacon Antenna'
  hak, /MESSAGE
  plot_graph, pby, '!6Power Received by Vertical Beacon Antenna'
  hak, /MESSAGE
  plot_graph, tot, '!6     Total Received Power at Beacon'
END
```

```
;author: Peter Dreisiger, Dec 1996 IE HFRD DSTO SALISBURY
;==================================================================
; This program generates power v's polarisation plots across beta(beacon) and
; beta(receiver) space. It also allows the user to analyse the probability of
; detecting signals with Xdb of the maximum detectable signal, and can view
; this data as either plots or animations
;==================================================================

PRO Receive_plot
  clear
  load_colours, 'FMS Rainbow'   ;define constants
  theta = findgen(360) /57.3
  cs = cos(theta)
  sn = sin(theta)
  vx = cs##cs
  vy = sn##sn
  hx = sn##cs
  hy = cs##sn
  a = [1, 2]
  lab = ['  0', ' 45', ' 90', '135', '180', '225', '270', '305', '360']
  thresh = 0.0
  steps = 0.0
  bet = 3.0

                                ;obtain parameters
  print & print & print & print & print
  PRINT, '==============================================================='
  read, bet, PROMPT='Please specify the maximum absolute value of BETA (dB): '
  read, steps, PROMPT='Enter the number of increments BETA will go through: '
  read, rbeta, PROMPT='Enter the magnitude of BETA for the receiver in dB: '
  PRINT, '==============================================================='
  print
  print, 'Please wait... Allocating memory'
  print
  bet = exp( -alog(10) /10 * bet)
  rbeta = 10^( -rbeta /10)
  power = fltarr(steps +1, 360, 360)
  prob = fltarr(steps +1, 2)
  window, TITLE='Passive Beacon Return Signal Analysis'

                                ;generate data slices
  print, 'Please wait... Generating data'
  print
  FOR BETA=0, steps DO BEGIN
    print_bar, beta, steps
    sbeta = (bet +(float(beta) *(1.0 -bet) ) /steps)^2
    power(beta, * , * ) = 10 * alog10((vx +sbeta* vy)^2 $
                                  + (rbeta *(sbeta * hx -hy))^2)
  ENDFOR

  choice = ''
  WHILE (choice NE '-1') DO BEGIN
    print &  print & print & print
    PRINT, '=============================================================
===' 
    print, 'Press 1  to view (theta,phi) at a specific value of Beta'
    print, '      2  to view the probability distributions for a specific S-N ra
tio'
    print, '      3  to view all (theta,phi) slides in an animation'
    print, '     -1 to quit'
    PRINT, '=============================================================
==='
    read, choice, PROMPT='Please enter your choice: '

    CASE choice OF
      '1':BEGIN                 ;view (theta,phi) plots
```



```
        beta = 0.0
        read, beta, PROMPT='Enter a value for Beta in dB (-1 to quit)'
        WHILE (beta GE 0) DO BEGIN
          beta = fix(steps /(1.0 -bet)*( -bet +10.0^( -beta /10.0)))
          IF (beta GT steps) THEN BETA=steps
          IF (beta LT 0) THEN beta = 0
          actb = -10 * alog10(bet +(float(beta) *(1.0 -bet) ) /steps )
          print, 'Retrieving slide for BETA=' +string(actb, FORMAT='(f4.2)')
          plot, a, /NODATA, POS=[98, 98, 460, 460], /DEVICE, TICKLEN= -0.02, $
           XTICKS=8, YTICKS=8, XMINOR=9, YMINOR=9, XTICKNAME=lab, $
           YTICKNAME=lab, XTITLE='!6Theta (degrees)', YTITLE='!6Phi (degrees)',
$
           TITLE='Power Distribution for Beta = ' +string(actb, FORMAT='(f4.2)')

          color_scale, /VERT, MINVAL= -20, INT=5, MAJ=4, XPOS=500, YPOS=100, $
           BAR_LENGTH=360, TITLE='S/N (dB)', MAX_USEABLE_COL=!colour.top
          tv, ie_bytscl(power(beta, *, * ), DMAX=0, DMIN= -20, $
                     TOP=!colour.top), 100, 100, /device
          beta = 0.0
          read, beta, PROMPT='Enter a value for Beta in dB (-1 to quit)'
        ENDWHILE
      END

      '2':BEGIN                 ;collating and displaying statistics
        read, thresh, PROMPT='Enter the cut-off ratio in dB: '
        print, 'Please wait... Collating statistics'
        print

        FOR BETA=0, steps DO BEGIN
          index = where(power(beta, * , * ) GT -thresh, count)
          prob(beta, 0) = -10 * alog10(bet +(float(beta) *(1.0 -bet) ) /steps )
          prob(beta, 1) = float(count) /129600
          print_bar, beta, steps
        ENDFOR
        erase
        baxis = -10 * alog10(bet +(1.0 -bet) * findgen(101) /100.0)
        plot, baxis, spline( -prob( *, 0), prob( *, 1), -baxis), XTITLE='!6Beta
' + $
         '(dB)', YTITLE='!6Probability', POS=[98, 98, 560, 460], /DEVICE
        read, thresh, PROMPT='Enter the cut-off ratio in dB (-1 to quit): '

        WHILE (thresh GE 0) DO BEGIN
        print, 'Please wait... Collating statistics'
        print
        FOR BETA=0, steps DO BEGIN
          index = where(power(beta, * , * ) GT -thresh, count)
          prob(beta, 0) = -10 * alog10(bet +(float(beta) *(1.0 -bet) ) /steps )
          prob(beta, 1) = float(count) /129600
          print_bar, beta, steps
        ENDFOR
        baxis = -10 * alog10(bet +(1.0 -bet) * findgen(101) /100.0)
        oplot, baxis, spline( -prob( *, 0), prob( *, 1), -baxis)
        read, thresh, PROMPT='Enter the cut-off ratio in dB (-1 to quit): '
        ENDWHILE
      END

      '3':BEGIN                 ;animate (theta,phi) plots
        plot, a, /NODATA, POS=[98, 98, 460, 460], /DEVICE, TICKLEN= -0.02, $
         XTICKS=8, YTICKS=8, XMINOR=9, YMINOR=9, XTICKNAME=lab, $
         YTICKNAME=lab, XTITLE='!6Theta (degrees)', YTITLE='!6Phi ' + $
         '(degrees)'
        color_scale, /VERT, MINVAL= -20, INT=5, MAJ=4, XPOS=500, YPOS=100, $
         BAR_LENGTH=360, TITLE='S/N (dB)', MAX_USEABLE_COL=!colour.top
       FOR BETA=0, steps DO BEGIN
        tv, ie_bytscl(power(beta, *, * ), DMAX=0, DMIN= -20, $
                   TOP=!colour.top), 100, 100, /device
```

```
        ENDFOR
      END

      Else:
    ENDCASE
  ENDWHILE
END
```

```
; author: Peter Dreisiger, Jan 1997
; Plots dP/df assuming a quasi-parabolic concentric ionosphere
; using effective frequency to take into account the magnetic
; fields


FUNCTION PolarRotation, freq, dist, bearing, fc
; author: Peter Dreisiger, HFRD Jan 1997
; Calculates polarisation rotation of a radio wave as it travels
; from alice springs to the beacon DIST kms away.
; ============================================================
; input:  freq    = operating frequency (MHz)
;         dist    = ground range (metres)
;         bearing = bearing (degrees east of north)
;         fc      = critical frequency (MHz)
; output: return  = polarisation angle (rads)
; ============================================================
  freq = double(freq)
  dist = double(dist)
  bearing = double(bearing)
  fc = double(fc)
  dip = 0d0
  decl = 0d0
  fH = 0d0
  po = 0d0
  px = 0d0
  dtor = atan(1d0) /45d0

  magnetic, 132.25, -17.9, dip, decl, fH
  fgm = double(0.5 * fH * cos(dip * dtor) * cos((decl -bearing) * dtor) )
  fRo = double((freq +fgm) /fc)
  fRx = double((freq -fgm) /fc)
  b0o = findb0(fRo, dist, findbmax(fRo) )
  b0x = findb0(fRx, dist, findbmax(fRx) )
  IF ((b0o NE -1) AND (b0x NE -1) ) THEN BEGIN
    po = double(phasepath(fRo, b0o) )
    px = double(phasepath(fRx, b0x) )
  ENDIF
  RETURN, atan(0d0, -1d0) * (po -px) * freq/300d0
END


FUNCTION FinddF, freq, fc, dist, bearing
; author: Peter Dreisiger, HFRD Jan 1997
; Calculates the change in operating frequency required to achieve
; a 90 degree rotation in polarisation angle at the receive site.
; ============================================================
; input:  freq    = operating frequency (MHz)
;         fc      = critical frequency (MHz)
;         dist    = ground range (metres)
;         bearing = bearing in degrees
; output: return  = polarisation bandwidth (MHz)
; ============================================================
  fmin = double(freq)
  fmax = fmin +2.5
  con = atan(1d0, 0d0)
  accuracy = 0.0001

  pbase = double(polarrotation(fmin, dist, bearing, fc) )
  pmin = double(0)
  pmax = double(polarrotation(fmax, dist, bearing, fc) -pbase)
  diff = fmax -fmin
  WHILE (diff GT accuracy)  DO BEGIN
    pmid = double(polarrotation(0.5 *(fmin +fmax), dist, bearing, fc) -pbase)
    IF ((con GE pmin) AND (con LT pmid) ) THEN fmax = 0.5 *(fmax +fmin)
    IF ((con GE pmid) AND (con LE pmax) ) THEN fmin = 0.5 *(fmax +fmin)
```



```
    pmin = double(polarrotation(fmin, dist, bearing, fc) -pbase)
    pmax = double(polarrotation(fmax, dist, bearing, FC) -pbase)
    diff = fmax -fmin
  ENDWHILE
  RETURN, (0.5 *(fmin +fmax) ) -freq
END


FUNCTION FinddD, dist, freq, fc, bearing
; author: Peter Dreisiger, HFRD Jan 1997
; Calculates the change in ground range required to achieve
; a 90 degree rotation in polarisation angle at the receive site.
; ============================================================
; input:  dist    = ground range (meters)
;         freq    = operating frequency (MHz)
;         fc      = critical frequency (MHz)
;         bearing = bearing in degrees
; output: return  = polarisation groundrange (meters)
; ============================================================
  dmin = double(dist)
  freq = double(freq)
  fc = double(fc)
  dmax = dmin +500000
  con = !Pi /2
  accuracy = 0.005
  pmin = double(polarrotation(freq, dmin, bearing, fc) )
  con = pmin -!pi /2
  pmax = double(polarrotation(freq, dmax, bearing, fc))
  diff = dmax -dmin
  WHILE (diff GT accuracy)  DO BEGIN
    pmid = double(polarrotation(freq, 0.5 * (dmin +dmax) , bearing, fc))
    IF ((con GE pmin) AND (con LT pmid) ) THEN dmin = 0.5 *(dmax +dmin)
    IF ((con GE pmid) AND (con LE pmax) ) THEN dmax = 0.5 *(dmax +dmin)
    pmin = double(polarrotation(freq, dmax, bearing, fc))
    pmax = double(polarrotation(freq, dmin, bearing, fc))
    diff = dmax -dmin
  ENDWHILE
  RETURN, (0.5 *(dmin +dmax) ) -dist
END


FUNCTION DpdF, freq, df, fc, dist
; author: Peter Dreisiger, HFRD Jan 1997
; Calculates the change in polarisation angle as a function of the
; change in operating frequency, using the linearised model.
; ============================================================
; input:  freq    = operating frequency (MHz)
;         df      = change in frequency (MHz)
;         fc      = critical frequency (MHz)
;         dist    = ground range (meters)
; ============================================================
  freq = double(freq)
  df = double(df)
  fc = double(fc)
  dist = double(dist)
  bearing = 0.0
  dtor = atan(1d0) /45d0
  magnetic, 132.25, -17.9, dip, decl, fH
  fgm = double(0.5 * fH * cos(dip * dtor) * cos((decl -bearing) * dtor) )

  fo = freq +fgm
  fx = freq -fgm
  b0o = findb0(fo /fc, dist, findbmax(fo /fc) )
  b0x = findb0(fx /fc, dist, findbmax(fx /fc) )
  po = phasepath(fo /fc, b0o)
  pdo = grouppath(fo /fc, b0o)
```

```
  px = phasepath(fx /fc, b0x)
  pdx = grouppath(fx /fc, b0x)
  RETURN, df * atan(0d0, -1d0) /300d0 *(pdo +px -po -pdx)
END


FUNCTION DfdP, freq, fc, dp, dist, bearing
; author: Peter Dreisiger, HFRD Jan 1997
; Calculates the change in frequency required to cause a change in
; polarisation angle of dP, using the linearised model.
; ==============================================================
; input:  freq    = operating frequency (MHz)
;         fc      = critical frequency (MHz)
;         dp      = change in polarisation angle (rads)
;         dist    = ground range (meters)
;         bearing = bearing (degrees east of north)
; ==============================================================
  freq = double(freq)
  fc = double(fc)
  dP = double(dP)
  dist = double(dist)
  bearing = double(bearing)
  magnetic, 132.25, -17.9, dip, decl, fH
  fgm = double(0.5 * fH * cos(dip * !dtor) * Cos((decl -bearing) * !dtor) )

  fo = freq +fgm
  fx = freq -fgm
  b0o = findb0(fo /fc, dist, findbmax(fo /fc) )
  b0x = findb0(fx /fc, dist, findbmax(fx /fc) )
  po = phasepath(fo /fc, b0o)
  pdo = grouppath(fo /fc, b0o)
  px = phasepath(fx /fc, b0x)
  pdx = grouppath(fx /fc, b0x)
  RETURN, 300d0 * dp /(atan(0d0, -1d0) *(pdo -po -pdx +px))
END


PRO TestError
; Test procedure used to evaluate the error in predicting a polarisation
; change of ANGLE using the linearised model
  angle = !pi /2
  bearing = 0.0
  cnt = 0
  err = fltarr(1001)
  smoth = fltarr(1001)
  FOR FREQ=20d0, 10d0, -0.01 DO BEGIN
    print_bar, cnt, 1000
    cnt = cnt +1
    df = dfdp(freq, 9, angle, 2000000, bearing)
    err((freq -10) * 100) = polarrotation(freq +df, 2000000, bearing, 9) $
                        - polarrotation(freq, 2000000, bearing, 9) -angle
  ENDFOR
  FOR I=2, 998 DO BEGIN
    smoth(i) = 0.2 * (err(i -2) +err(i -1) +err(i) +err(i +1) +err(i +2) )
  ENDFOR

  stop
END


PRO Distband
; Procedure to calculate the polarisation ground range
  fc = double(8.97770275)
  b = fltarr(361, 100)
  read, freq, PROMPT='Enter operating frequency: '
  freq = double(freq)
```



```
  FOR DIST=1500000d0, 2500000d0, 10000 DO BEGIN
    print_bar, (dist -1500000) /10000, 100
    fr = 0.8d0 * sqrt(1d0 +(dist /grouppath(0.8, findb0(0.8, dist, $
                                 findbmax(0.8) ) ) ) ^2)
    FOR BEAR= -45d0, 45d0, 0.05 DO BEGIN
      b((bear +45) * 4, (dist -1500000) /10000) = abs(cos(polarrotation(fr * fc,
 $
                              dist, bear, fc) ) )
    ENDFOR
  ENDFOR
stop
END


PRO Polarband
; Plots the polarisation bandwidth as a function of frequency
; for a given bearing.
  fc = double(8.97770275)
  b = dblarr(21, 5)
  bearing = 0.0
  FOR FREQ=double(10), 20, 0.5 DO BEGIN
    print, freq
    b((freq -10) * 2, 0) = dfdp(freq, fc, !pi /2, 1750000, 0)
    b((freq -10) * 2, 1) = dfdp(freq, fc, !pi /2, 1750000, 15)
    b((freq -10) * 2, 2) = dfdp(freq, fc, !pi /2, 1750000, 30)
    b((freq -10) * 2, 3) = dfdp(freq, fc, !pi /2, 1750000, 45)
    b((freq -10) * 2, 4) = dfdp(freq, fc, !pi /2, 1750000, 60)
  ENDFOR
  stop
END
```

```
FUNCTION Findb0, fR, dist, beta0max
; author: Peter Dreisiger, IE HFRD DSTO 96-97
; ----------------------------------------------------------
; This function returns the value of b0 for which a ray
; of frequency fR=f/fc will cover a ground distance of
; dist. beta0max is a parameter found using 'findbmax'
; that specifies at what elevation angle the high-angle
; ray commences.
; If the specified ground range is not possible at the
; given frequency, -1 is returned.
  fR = double(fR)
  dist = double(dist)
  beta0max = double(beta0max)
  accuracy = 0.002
  bbot = double(0.0)
  btop = beta0max
  dmx = groundrange(fR, bbot)
  dmn = groundrange(fR, btop)
  diff = dmx -dmn

  IF ((dist GE dmn) AND (dist LE dmx) AND (beta0max GE 0) ) THEN BEGIN
    WHILE (diff GT accuracy) DO BEGIN
      dmd = groundrange(fR, 0.5 *(bbot +btop) )
      diff = abs(dist -dmd)
      IF ((dist GE dmn) AND (dist LE dmd) ) THEN bbot = 0.5 *(btop +bbot)
      IF ((dist GE dmd) AND (dist LE dmx) ) THEN btop = 0.5 *(btop +bbot)
      dmx = groundrange(fR, bbot)
      dmn = groundrange(fR, btop)
    ENDWHILE
    b0 = 0.5 *(bbot +btop)
  ENDIF ELSE BEGIN
    b0 = -1
  ENDELSE
  RETURN,  b0
END


FUNCTION Findbmax, fR
; author: Peter Dreisiger, IE HFRD DSTO 96-97
; ----------------------------------------------------------
; This function returns the maximum value of b0 for which the
; ray is a low-angle wave
  fR = double(fR)
  last = double(groundrange(fR, 0) )
  IF (last GT 0.0) THEN BEGIN
    current = double(last)
    beta = 0.0
    betamax = 90 * !dtor
    WHILE ((beta LE 90) AND (BETAMAX EQ 90 * !dtor) ) DO BEGIN
      current = groundrange(fR, beta * !dtor)
      IF (current GT last) THEN betamax = double(beta * !dtor)
      last = current
      beta = beta +0.5
    ENDWHILE
  ENDIF ELSE BEGIN
    betamax = -1
  ENDELSE
  RETURN, betamax
END


FUNCTION GroundRange, fR, b0
; author: Peter Dreisiger, IE HFRD DSTO 96-97
; ----------------------------------------------------------
; Uses Croft's solution for ray propagation in a QP concentric
; ionosphere to determine the ground range for a given frequency
```



```
; ratio (f/fc) and elevation angle.
  ro = double(6370000)
  rm = double(ro +300000)
  ym = double(100000)
  rb = double(rm -ym)
  fR = double(fR)
  b0 = double(b0)
  d = 0.0

  a = 1.0 -1.0 /fR^2 +(rb /(fR * ym) ) ^2
  b = -2.0 * rm *(rb /(fR * ym) ) ^2
  c = (rb * rm /(fR * ym) ) ^2 -(ro * cos(b0) ) ^2
  IF (((b^2 -4 * a * c) GE 0) AND (b0 GE 0)) THEN BEGIN
  g = acos(ro /rb * cos(b0) )
    d = 2 * ro *((g -b0) -ro * cos(b0) * 0.5 /sqrt(c) * alog((b^2 -4 * a * c) $
                  /(4.0 * c *(sin(g) +sqrt(c) /rb +0.5 * b /sqrt(c) )^2) ) )

  ENDIF
  IF (d LT 0) THEN d = 0
  RETURN, double(d)
END


FUNCTION Grouppath, fR, b0
; author: Peter Dreisiger, IE HFRD DSTO 96-97
; ----------------------------------------------------------
; Uses Croft's solution for ray propagation in a QP concentric
; ionosphere to determine the group path for a given frequency
; ratio (f/fc) and elevation angle.
  ro = double(6370000)
  rm = double(ro +300000)
  ym = double(100000)
  rb = double(rm -ym)
  fR = double(fR)
  b0 = double(b0)
  p = 0.0

  a = 1.0 -1.0 /fR^2 +(rb /(fR * ym) )^2
  b = -2.0 * rm *(rb /(fR * ym) )^2
  c = (rb * rm /(fR * ym) )^2 -(ro * cos(b0) )^2
  g = acos(ro /rb * cos(b0) )
  IF ((b^2 -4 * a * c)  GE 0) THEN $
   p = 2 *(rb * sin(g) -ro * sin(b0) +1 /a *( -rb * sin(g) -0.25 * b /sqrt(a) $
              * alog((b^2 -4 * a * c) /(2 * a $
                          * rb +b +2 * rb * sqrt(a) * sin(g) )^2 ) ) )
  RETURN, double(p)
END


FUNCTION Phasepath, fR, b0
; author: Peter Dreisiger, IE HFRD DSTO 96-97
; ----------------------------------------------------------
; Uses Croft's solution for ray propagation in a QP concentric
; ionosphere to determine the phase path for a given frequency
; ratio (f/fc) and elevation angle.
  ro = double(6370000)
  rm = double(ro +300000)
  ym = double(100000)
  rb = double(rm -ym)
  fR = double(fR)
  b0 = double(b0)
  p = 0.0

  a = 1.0 -1.0 /fR^2 +(rb /(fR * ym) )^2
  b = -2.0 * rm *(rb /(fR * ym) )^2
  c = (rb * rm /(fR * ym) )^2 -(ro * cos(b0) )^2
```

```
  g = acos(ro /rb * cos(b0) )
  IF ((b^2 -4 * a * c)  GE 0) THEN $
   P=2.0 *( -ro * sin(b0) +b /4.0 *(1.0 /sqrt(a) * alog((b^2 -4.0 * a * c) $
                      /(4.0 *(a * rb +b /2.0 +sqrt(a) * rb * sin(g) )^2) ) $
                       +rm /sqrt(c) * alog((b^2 -4.0 * a * c) /(4.0 * c *( $
                           sin(g) +sqrt(c) /rb +0.5 * b /sqrt(c) )^2 ) ) ) )
  RETURN, double(p)
END


PRO Magnetic, Glong, Glat, Dip, Decl, fH
; author: Chris Coleman, IE HFRD DSTO 94
; --------------------------------------------------------
; Calculates the magnetic field wuantites for an offset dipole
; Glat = mean geographic latitude
; glong = mean geographic longitude
;
; returns Dip, Decl, fH
;
; Fgm=0.5*fH*cos(dip*!dtor)*cos(decl-bearing)*!dtor)
  scale = 45.0 /atan(1.0)
  Dlong = double(295.75)
  Dlat = double(77.75)
  SGlat = sin(Glat /scale)
  SDlat = sin(Dlat /scale)
  CGlat = cos(Glat /scale)
  CDlat = cos(Dlat /scale)
  Clong = cos((Glong -Dlong) /scale)
  Slong = sin((Glong -Dlong) /scale)
  SMlat = SGlat * SDlat +CGlat * CDlat * Clong
  CMlat = sqrt(abs(1.0 -SMlat^2) )
  Dip = scale * atan(SMlat, CMlat)
  glo = -CDlat * Slong
  gla = CGlat * SDlat -SGlat * CDlat * Clong
  Decl = scale * atan(glo, gla)
  fH = 0.73 * sqrt(1.0 +3.0 * SMlat^2)
END
```